Research Article

Adaptive beamformers using 2D-novel ULA for cellular communication

Veerendra Dakulagi[1] · Mukil Alagirisamy[1]

Abstract

This paper presents the use of a novel two dimensional (2D) uniform linear array for adaptive beamforming algorithms. We used this novel antenna configuration to study the popular adaptive beamforming algorithms, namely, least mean square algorithm (LMS) and least normalized mean square algorithm (NLMS) and also proposed a variable step-size NLMS (VSSNLMS) algorithm. These beamformers are named as 2D-LMS, 2D-NLMS and 2D-VSSNLMS algorithms respectively. These algorithms not only improve the convergence rate but also mitigates the effect of interference by producing deep nulls in the interference directions. These algorithms are suitable for low-power, low-cost, anti-interference MIMO-WLAN, WiMAX, 4G LTE and other advanced communication systems.

Keywords 4G LTE · LMS · MIMO · NLMS · WiMAX · ULA

1 Introduction

The shape of radiation pattern of smart antenna system is controlled through the use of beamforming algorithms. It is based on a certain criteria that may be enhancing the signal-to-interference ratio (SIR), minimizing mean square error (MSE), minimizing the variance beam steering toward signal of interest (SOI), tracking a moving emitter and mitigating the interference signals. These algorithms can be implemented electronically via analog devices. The implementations will be easier and accurate using digital signal processor. An antenna radiation pattern (also called as beam) formed by digital signal processor is known as digital beamforming. Figure 1 shows the analog (a) and digital beamforming (b) structures.

1.1 Least mean square algorithm (LMS)

The LMS algorithm to estimate the complex weights of an antenna array is widespread. The LMS algorithm is shown in Fig. 2 is a gradient-based method. The convergence rate of this algorithm is completely depends on the step size (μ) parameter. If it is too small, convergence rate of the algorithm will be too slow and it results into the over damped case. If μ is too large, then we will have under damped case. Hence, the convergence rate of an algorithm is very important aspect and its practical importance for cellular communication is reported in [1].

The standard LMS algorithm uses fixed step size. In [2], it is shown that the step size is varied during each iteration to improve the convergence speed by minimizing the MSE. The convergence speed can be increased for broadband signals by implementing LMS algorithm in frequency domain. This concept is studied in detail in [3–5]. It reduces the computational complexity of algorithm.

The sign LMS algorithm is presented in [6]. In this work the error signal between the reference signal and antenna array output is replaced completely by its sign. Hence, this method is called as sign LMS and its computational complexity is less than the normal LMS algorithm. Practical application of LMS algorithm for cellular communications using an antenna array is studied for indoor radio

Veerendra Dakulagi, veerendra.gndec@gmail.com | [1]Department of Electronics and Communication Engineering, Lincoln University College, Petaling Jaya, Malaysia.

SN Applied Sciences (2019) 1:1001 | https://doi.org/10.1007/s42452-019-1009-z

Received: 8 April 2019 / Accepted: 29 July 2019 / Published online: 8 August 2019

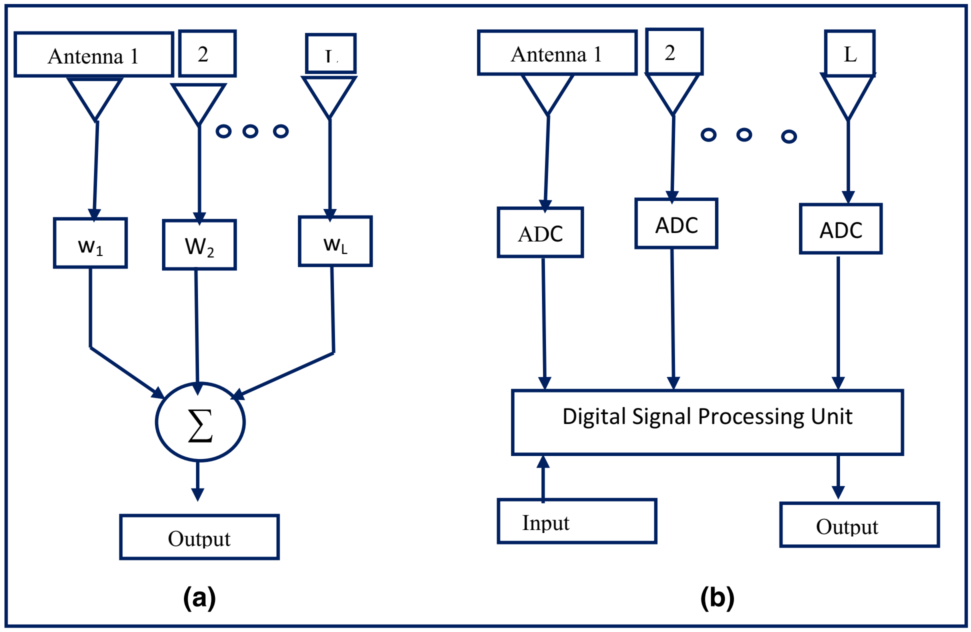

Fig. 1 **a** Analog beamforming, **b** digital beamforming

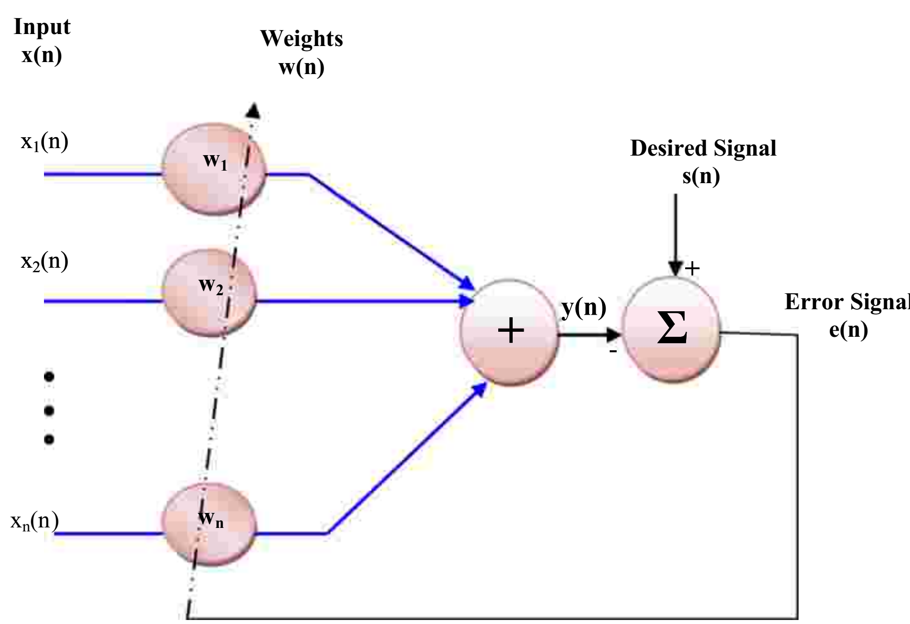

Fig. 2 LMS beamformer

systems in [7], mobile communication systems in [8] and satellite-to-satellite systems in [9].

1.2 Normalized LMS (NLMS) algorithm

In NLMS algorithm, for each iteration, it uses a data-dependent step size. The step size of this algorithm at *n*th iteration is given by $\mu(n) = \frac{\mu}{x^{H}(n)x(n)}$. Here, the value of μ is constant. The standard LMS algorithm requires the calculation of eigenvalues for covariance matrix. This is avoided in NLMS algorithm. NLMS has better convergence speed as compared to normal LMS algorithm. Normalizing makes it less sensitive as compared LMS algorithm [10, 11]. Application of NLMS algorithm for mobile communications is reported in [12]. Unbiased impulse response-based least mean square algorithm is presented in [13]. Performance of RLS and LMS is analyzed and a new method which combines both the algorithms is called RLS adaptive beamformer has been devised in [14–16]. RLS algorithm has better convergence rate as compared to LMS algorithm, but its computational complexity for multiple input multiple output (MIMO) is large. A new approach called MIR-LMS algorithm for an adaptive array applicable for high-speed mobile communication has been reported in [17–19]. The re-weighted l_1-norm penalized scheme for LMS algorithm is presented in [20]. This method is studied in detail for estimation of sparse channel, and its significance for wireless communication application is discussed. This algorithm has better performance as compared to standard LMS algorithm in terms of convergence speed. In most of the above methods [21–27], less importance is given to avoid the interferences, which is one of the most important requirements of practical applications.

Hence in this work, a novel 2D-ULA which consists of horizontal–vertical ULA configuration is deployed for LMS and NLMS algorithm and proposed VSSNLMS to mitigate the effect of interferences.

2 Array signal model

Consider a geometry of 2D-ULA with $2L$ antenna elements distributed horizontally and vertically with $m(m<L)$ number of source signals as shown in Fig. 3. Let the spacing between each antenna element be $d = \lambda/2$. Here $\lambda = v/f$, where v is the speed of light and f is the frequency of received signals, respectively. Let the received signal $x(n)$ impinges 2D-ULA with directions $\theta\left(\theta_1, \theta_2, \dots, \theta_m\right)$ in the far field.

2.1 Signal model of 1-D ULA

The induced signal in noisy environment at nth time can be expressed as

$$\mathbf{x}(n) = \mathbf{s}(n)\mathbf{a}\left(\theta_o\right) + \sum_{i=1}^{m} i_i(n)\mathbf{a}\left(\theta_i\right) + \mathbf{n}_o(n) \quad (1)$$

where $\mathbf{s}(n)$ is the desired signal, i_n is the interference signal, $\mathbf{a}\left(\theta_o\right)$ is the steering vector of desired signal, $\mathbf{a}\left(\theta_i\right)$

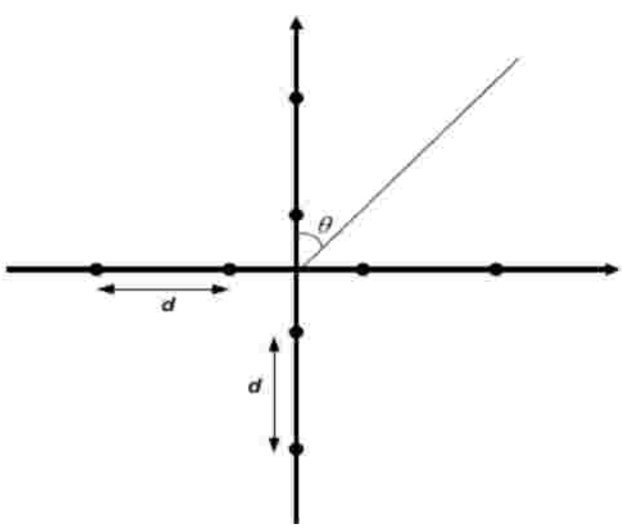

Fig. 3 Considered 2D-ULA configuration

is the steering vector of ith interference signal, $\mathbf{n}(n)$ is the White Gaussian noise with zero mean and m is the number of jamming (interference) sources

The above equation can be expressed in matrix form as:

$$\mathbf{X} = A_o\mathbf{S} + A_i\mathbf{I} + \mathbf{N} \quad (2)$$

where $\mathbf{X} = L \times K$ matrix of induced signal. Here, K is the number of snapshots, $\mathbf{S}$ = reference signal matrix, $A_o = L \times 1$ steering vector of desired signal, $A_i = L \times 1$ steering vector obtained by array manifold vector (all columns), $\mathbf{I} = 1 \times K$ interfering sample signal, $\mathbf{N} = L \times K$ Gaussian noise with zero mean and σ_n^2 variance.

The expression for the array vector can be written as:

$$A = \begin{bmatrix} 1 \\ e^{jkd\sin\theta_1} \\ \vdots \\ e^{jkd(L-1)\sin\theta_1} \end{bmatrix} + \begin{bmatrix} 1 \\ e^{jkd\sin\theta_2} \\ \vdots \\ e^{jkd(L-1)\sin\theta_2} \end{bmatrix} + \cdots + \begin{bmatrix} 1 \\ e^{jkd\sin\theta_m} \\ \vdots \\ e^{jkd(L-1)\sin\theta_m} \end{bmatrix} \quad (3)$$

Here, the components which are summed are the $L \times 1$ array manifold vectors and this can be denoted as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{jkd\sin\theta} \\ \vdots \\ e^{jkd(L-1)\sin\theta} \end{bmatrix} = \left[1, e^{jkd\sin\theta}, \dots, e^{jkd(L-1)\sin\theta}\right]^{T} \quad (4)$$

Here, $(\cdot)^T$ is the transpose;$(\cdot)^H$ denotes the Hermitian transpose throughout this paper.

The array factor (AF) and it can be expressed as

$$\mathrm{AF} = \left[1, e^{jkd\sin\theta}, \dots, e^{jkd(L-1)\sin\theta}\right] \quad (5)$$

This AF is a very important parameter to obtain the radiation pattern of an antenna array.

The beam steering array weights $w(n)$ can be obtained by solving the Wiener–Hopf equation as below:

$$w(n) = \mathbf{R}_{xx}^{-1} r_{sx} \quad (6)$$

where $\mathbf{R}_{xx}$ the autocorrelation of arriving signal $x(n)$, r_{sx} is the cross-correlation between the induced signal $x(n)$ and a reference signal $s(n)$.

In practice, designing an adaptive beamformer is much difficult than the conventional non-adaptive beamformer using the Wiener–Hopf method [2]. This problem can be solved by using the weight update equation, and it can be expressed for adaptive beamformer as

$$w(n+1) = w(n) + \Delta w(n) \quad (7)$$

where $\Delta w(n)$ is the correlation applied to obtain the new weights. The computation weights can vary from beam steering algorithms to algorithms. Thus, the adaptive beam steering uses the weight update equation to form main beam in the look direction and the nulls in the direction of interferences.

2.2 Signal model of 2D-ULA

The geometry of 2D-ULA shown in Fig. 3 uses doubly crossed horizontal and vertical ULAs. This configuration when used in beamforming algorithm gives better angular resolution and deep nulls as compared to the conventional ULA. Hence, this method has more interference rejection capability as compared to the ULA-based beamformers.

Let us recall (1). That is

$$\mathbf{x}(n) = \mathbf{s}(n) + \mathbf{i}(n) + \mathbf{n}(n) = \mathbf{a}(\theta_0)\mathbf{s}(n) + i(n)\sum_{i=1}^{m} \mathbf{a}(\theta_i) + \mathbf{n}(n) \tag{8}$$

Now, let us apply this equation (concept) to the 2D-ULA configuration. The received narrow-band signal for the 2D-ULA can be expressed as

$$\begin{aligned}\mathbf{x}(n) &= \mathbf{x}_{\mathrm{H}}(n) + \mathbf{x}_{\mathrm{V}}(n)\\ &= \left[\mathbf{a}(\theta_0)\mathbf{s}(n) + \mathbf{i}(n)\sum_{i=1}^{m} \mathbf{a}(\theta_i) + \mathbf{n}(n)\right]_{\mathrm{H}} + \left[\mathbf{a}(\theta_0)\mathbf{s}(n) + \mathbf{i}(n)\sum_{i=1}^{m} \mathbf{a}(\theta_i) + \mathbf{n}(n)\right]_{\mathrm{V}}\end{aligned} \tag{9}$$

$$= \left[\sum_{m=1}^{M} \mathbf{a}_{\mathrm{H}}(\theta_m) + \mathbf{a}_{\mathrm{V}}(\theta_m)\right]\mathbf{s}_m(t) \tag{10}$$

Here, $\mathbf{a}_{\mathrm{H}}(\theta_m) = L \times 1$ is steering vector of horizontal ULA, $\mathbf{a}_{\mathrm{V}}(\theta_m) = L \times 1$ steering vector of vertical ULA.

The new expressions for the horizontal and vertical array steering vectors are expressed as

$$\mathbf{a}_{\mathrm{H}}(\theta_{\mathrm{m}}) = \left[\mathrm{e}^{\{j(n-1)2\pi(d/\lambda)\}\sin\theta_m}\right]^{\mathrm{T}} \tag{11}$$

$$\mathbf{a}(\theta_m) = \left[\mathrm{e}^{\{j(n-1)2\pi(d/\lambda)\}\cos\theta_m}\right]^{\mathrm{T}} \tag{12}$$

Now the array factor(AF) for 2D-ULA can be expressed as

$$\mathrm{AF} = [\mathrm{AF}]_{\mathrm{H}} + [\mathrm{AF}]_{\mathrm{V}} \tag{13}$$

$$\text{Here}: \quad [\mathrm{AF}]_{\mathrm{H}} = \left(\sum_{i=1}^{L} w^{\mathrm{H}}(i)\ \mathrm{e}^{j2\pi d\sin(\theta)}\right)_{\mathrm{H}} \tag{14}$$

$$[\mathrm{AF}]_{\mathrm{V}} = \left(\sum_{i=1}^{L} w^{\mathrm{H}}(i)\ \mathrm{e}^{j2\pi d\cos(\theta)}\right)_{\mathrm{V}} \quad \text{Here}: -90° \le \theta + 0.001 \le +90° \tag{15}$$

The array output can be expressed as

$$\mathbf{y}(n) = w^{\mathrm{H}} \cdot \mathbf{x}(n) \tag{16}$$

We have deployed this novel method to LMS, NLMS and proposed VSSNLMS algorithms to study their performance in various conditions.

3 Proposed adaptive beamforming algorithms

In this section, firstly, a novel variable step-size normalized least mean square (VSSNLMS) algorithm is presented. Then, the LMS, the NLMS and the proposed VSSNLMS algorithm is studied using novel 2D-ULA.

3.1 Novel VSSNLMS algorithm

The main objective of the proposed VSSNLMS algorithm is that it uses variable step in the NLMS algorithm. This solves the trade-off issue between the convergence rate and the steady-state MSE.

The convergence rate of this algorithm can be speeded up by using a larger step size in the initial stages, and the optimum value can be obtained by using a smaller step size near the steady-state MSE. The VSSNLMS algorithm is developed using the following weight updating equation:

$$w(n+1) = w(n) + \frac{\mu(n)\,e(n)\,x(n)}{\sigma + \|x(n)\|^2} \tag{17}$$

where,

$$\mu(n) = \begin{matrix} \left\{\left(\frac{6}{L}\right)^2\left(j\left(\frac{6}{L}\right)^2\right) + 0.001\right\}, & 1 \le j \le \frac{L}{6} \\ 0.0001 & \frac{L}{6} \le \mathrm{j} \le L \end{matrix} \tag{18}$$

$w(n+1)$ = updated array weights ,
$w(n)$ = previous weights , $x(n)$ = received signal,
$e(n) = s(n) - x(n)$ = error signal , σ = variance = 0.01.

3.2 Proposed novel 2D-LMS algorithm

Novel 2D-LMS algorithm can be implemented using the following weight updating equation:

$$w(n+1) + w(n) + \mu e^*(n)x(n) \tag{19}$$

$$\begin{aligned}&\text{Here}: \mathbf{x}(n) = \mathbf{x}_{\mathrm{H}}(n) + \mathbf{x}_{\mathrm{V}}(n) = \left[\sum_{m=1}^{M} \mathbf{a}_{\mathrm{h}}(\theta_m) + \mathbf{a}_{\mathrm{V}}(\theta_m)\right]\mathbf{s}_m(t)\\ &\mu = 0.01(\text{fixed value})\end{aligned} \tag{20}$$

The array output is expressed as $\mathbf{y}(n) = w(n) * \mathbf{x}(n)$

3.3 Novel 2D-NLMS algorithm

Novel 2D-NLMS algorithm can be implemented using the following weight updating equation:

$$w(n+1) = w(n) + \frac{\mu(n)\, e(n)\, x(n)}{\sigma + \|x(n)\|^2}; \tag{21}$$

$$\mathbf{x}(n) = x_{\mathrm{H}}(n) + \mathbf{x}_{\mathrm{V}}(n) = \left[\sum_{m=1}^{M} \mathbf{a}_{\mathrm{H}}(\theta_m) + \mathbf{a}_{\mathrm{V}}(\theta_m)\right] \mathbf{s}_m(t) \tag{22}$$

$w(n+1)$ = updated array weights
$w(n)$ = previous weights , μ, = 0.01(fixed value).

The array output is expressed as $\mathbf{y}(n) = w(n) * \mathbf{x}(n)$

3.4 Novel 2D-VSSNLMS algorithm

Let us recall the weight updating equation of VSSNLMS algorithm

$$w(n+1) = w(n) + \frac{\mu(n)\, e(n)\, x(n)}{\sigma + \|x(n)\|^2} \tag{23}$$

where

$$\mathbf{x}(n) = \mathbf{x}_{\mathrm{H}}(n) + \mathbf{x}_{\mathrm{V}}(n) = \left[\sum_{m=1}^{M} \mathbf{a}_{\mathrm{H}}(\theta_m) + \mathbf{a}_{\mathrm{V}}(\theta_m)\right] \mathbf{s}_m(t) \tag{24}$$

$$\mu(n) = \left\{ \left(\frac{6}{L}\right)^2 \left(j\left(\frac{6}{L}\right)^2\right) + 0.001 \right\},\ 1 \le j \le \frac{L}{6}$$
$$0.0001 \qquad \frac{L}{6} \le \mathrm{j} \le L$$

Finally, the output of antenna array can be expressed as

$$\mathbf{y}(n) = w(n)^{\mathrm{T}} \mathbf{x}(n) \tag{25}$$

and the equation of array factor is : $\mathrm{AF} = [\mathrm{AF}]_{\mathrm{H}} + [\mathrm{AF}]_{\mathrm{V}}$ (26)

4 Results and discussion

In this section, firstly, the results of proposed VSSNLMS algorithm are discussed. Then, in the later part of this section, 2D-Novel ULA configuration is deployed to study its behavior. Finally, the results are compared with popular LMS and NLMS algorithms.

4.1 Results of VSSNLMS algorithm

Let us consider a ULA with $L = 10$ antenna elements separated by $d = \lambda/2$. Let the desired signal impinges the ULA at 0° and let the directions of interferences be [– 20° 0° 30°]. Let the value of step size be adaptive. The radiation pattern of proposed VSSNLMS algorithm is a plot of array factor versus angle of arrival of desired signal is shown in Fig. 4. And the convergence analysis is presented in Fig. 5. The computer results clearly show that the proposed method has narrow beam, representing high directivity and also it requires less number of iterations for proper beamforming. Furthermore, the beam of proposed VSSNLMS algorithm is steered in different directions [i.e., for $\theta = -10°$, 0°, – 40° and – 60°], which is shown in Fig. 6. The performance of proposed algorithm is compared with LMS and NMLS algorithms in all examples.

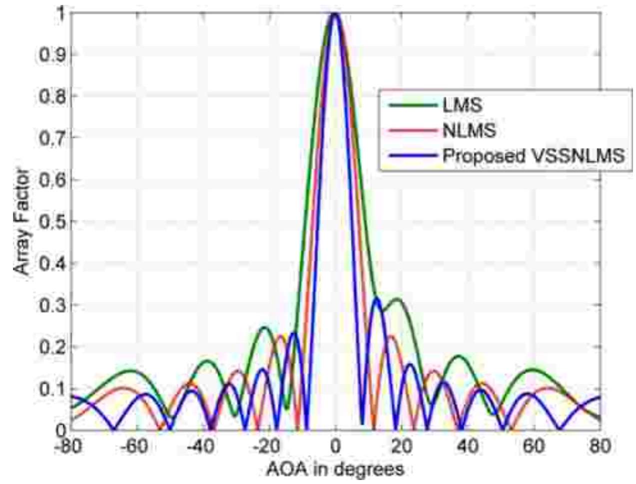

Fig. 4 Radiation pattern of LMS, NLMS and VSSNLMS

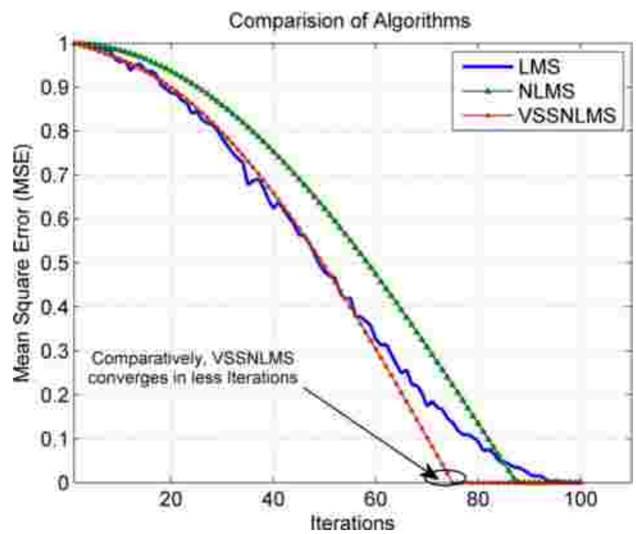

Fig. 5 Convergence analysis of LMS, NLMS and VSSNLMS

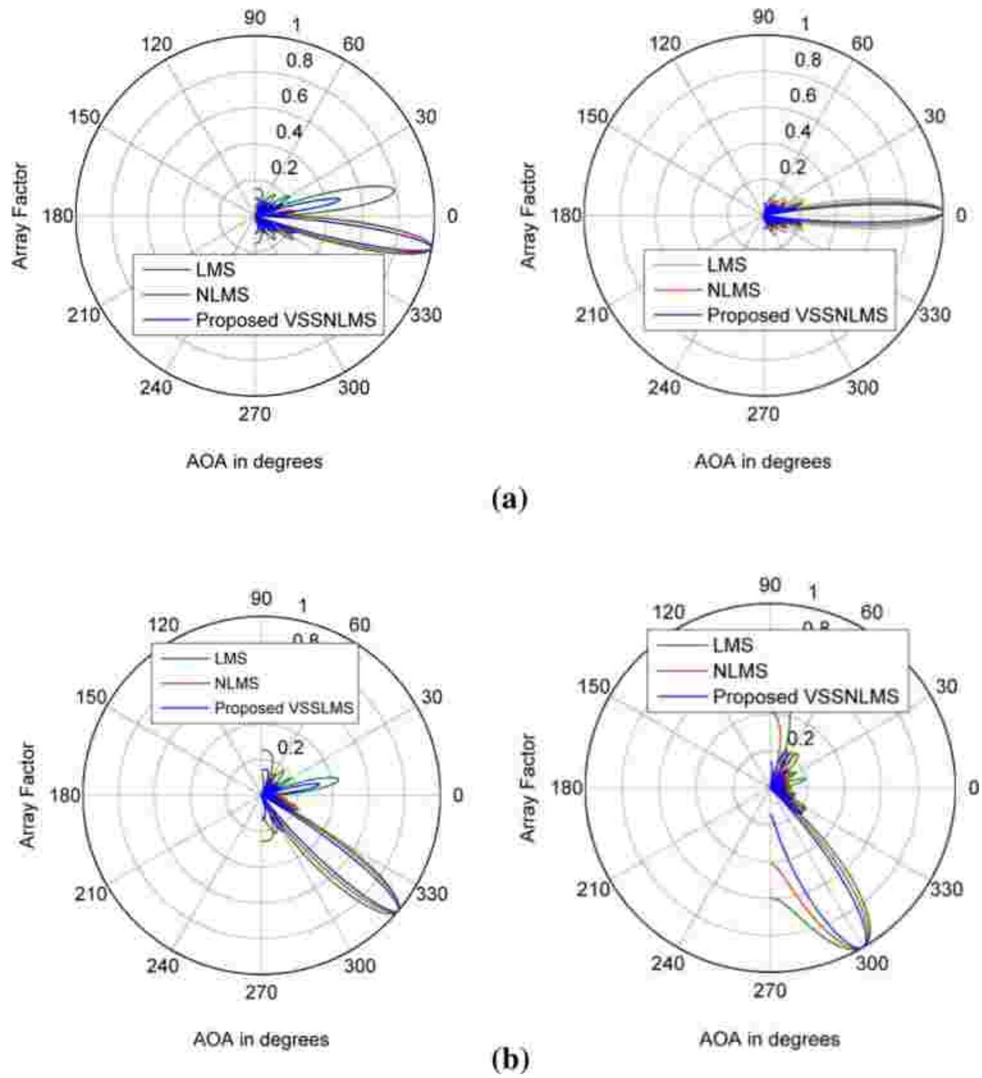

Fig. 6 Beam steering of adaptive algorithms for **a** $\theta = -10°$ and 0° and **b** θ = −40° and −60°

The standard LMS algorithm requires more than 100 iterations for the satisfactory performance which corresponds to almost the half cycle of signal of interest (SOI). Due to this, the LMS algorithm is not suitable for many wireless communication applications, particularly for 4G LTE, 5G and beyond.

Hence, this problem is solved by using a variable step-size normalized least mean square (VSSNLMS) algorithm. Experimental results (Fig. 5) clearly show that proposed VSSNLMS converges fast as compared to classical LMS and NLMS methods. Furthermore, we studied the LMS, NLMS and proposed VSSNLMS using novel 2D-ULA structure to mitigate the effect of interferences.

4.2 Adaptive beamforming algorithms using 2D-novel ULA

Let us consider a 2D-novel ULA with number of antennas $L = 10$, spacing between the antennas be $d = 0.5\lambda$, let the direction of arrival (DOA) of desired signal be $\theta = 0°$, SNR = 20 dB, let the directions of six interferences be [−60°, −40° and 60°]. Let us assume that the noise is white Gaussian with σ^2 variance and zero mean. In these simulations, variable step size is used for proposed 2D-VSSNLMS algorithm and fixed step size is used for the remaining methods. Figures 7, 8 and 9 show the beam patterns of 2D-LMS, 2D-NLMS and 2D-VSSNLMS, respectively.

The normalized array factors of the proposed method using 2D-novel ULA and the classical methods using normal ULA when the average depth of nulls are at 0° and 60° are tabulated in the Tables 1 and 2 respectively. Here, the normalized array factors of the LMS, NLMS, RLS, SMI, LMS/SMI and NLMS/RLS algorithms are taken directly from [21, 22]. The graphical representations of above tables are depicted in Figs. 10 and 11 respectively.

From the computer simulations and Tables 1 and 2, we note that, as compared to the normal ULA-based algorithms, the proposed 2D-VSSNLMS gives deep nulls and

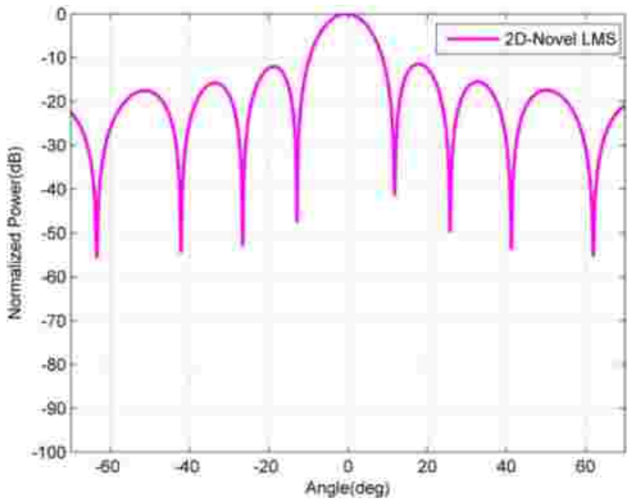

Fig. 7 Beam pattern of 2D-LMS

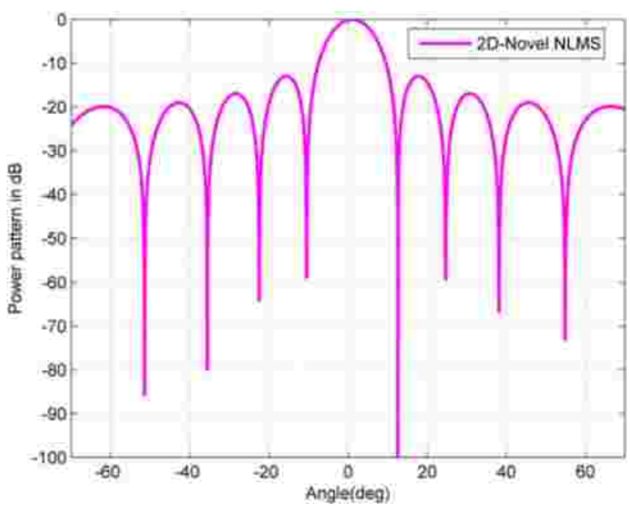

Fig. 8 Beam pattern of 2D-NLMS

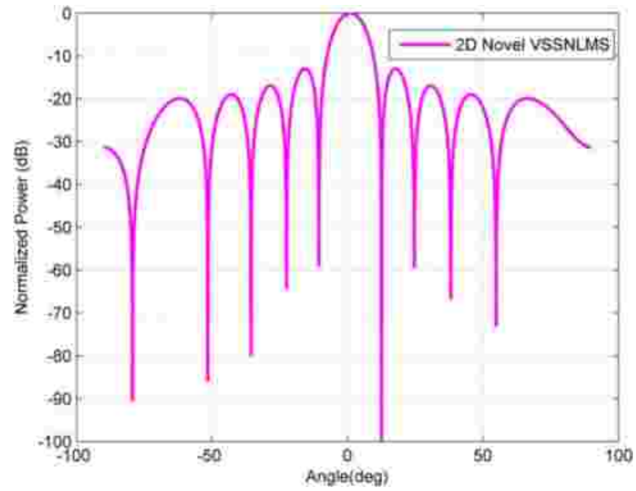

Fig. 9 Beam patterns of 2D-VSSNLMS

hence is more immune to noise. The negative aspect of 2D algorithms is that they demand fast analog to digital (ADC) converters which might lead to high sampling rate. Also, these algorithms are not suitable to resolve wide-band signals.

5 Conclusion

Adaptive beamformers using 2D-ULA structure mitigates the effect of interference by producing deep nulls. Computer simulations are evident for the effectiveness of proposed methods. The proposed 2D-VSSNLMS algorithm requires less number of iterations for beamforming and more immune to interferences. This makes it suitable for low-power, low-cost anti-interference MIMO-WLAN, WiMAX, 4G LTE and other advanced communication systems. The computer simulations have demonstrated the effectiveness and superiority of the proposed algorithms.

Table 1 Performance of beamforming algorithms using ULA and 2D-ULA at DOA = 0°

N	2D-VSSNLMS	NLMS/RLS	LMS/SMI	RLS	SMI	NLMS	LMS
8	−74	−56.73	−53.17	−55.39	−58.99	−43.57	−48.36
16	−81	−58.87	−52.78	−57.35	−60.52	−46.93	−50.11
24	−84	−60.87	−50.89	−59.02	−61.43	−49.33	−50.15
32	−87	−62.45	−48.46	−59.18	−60.72	−50.14	−48.79
51	−90	−63.84	−55.09	−59.6	−61.65	−51.57	−47.35

Table 2 Performance of beamforming algorithms using ULA and 2D-ULA at DOA = 60°

N	2D-VSSNLMS	NLMS/RLS	LMS/SMI	RLS	SMI	NLMS	LMS
8	−68	−55.74	−53.74	−53.21	−59.11	−43.11	−49.11
16	−72	−59.25	−52.15	−58.55	−62.15	−46.75	−49.65
24	−73	−60.22	−51.42	−60.12	−63.72	−48.42	−50.2
32	−74	−63.92	−50.52	−62.82	−63.82	−49.16	−48.86
51	−80	−63.61	−55.61	−63.41	−65.11	−53.51	−48.55

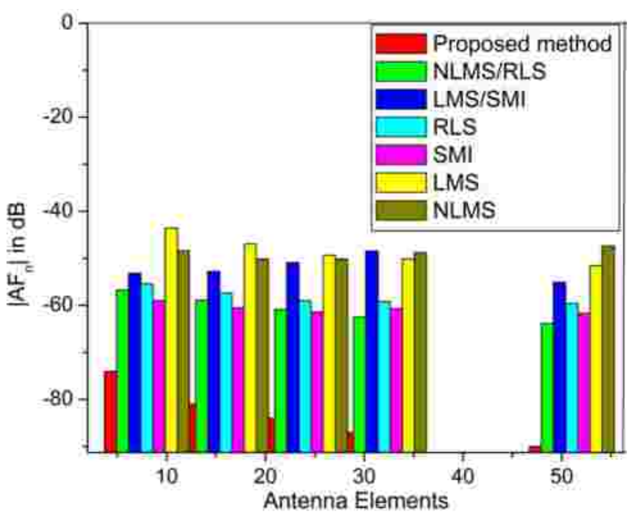

Fig. 10 Average nulls depth in decibels at 0° versus antenna elements

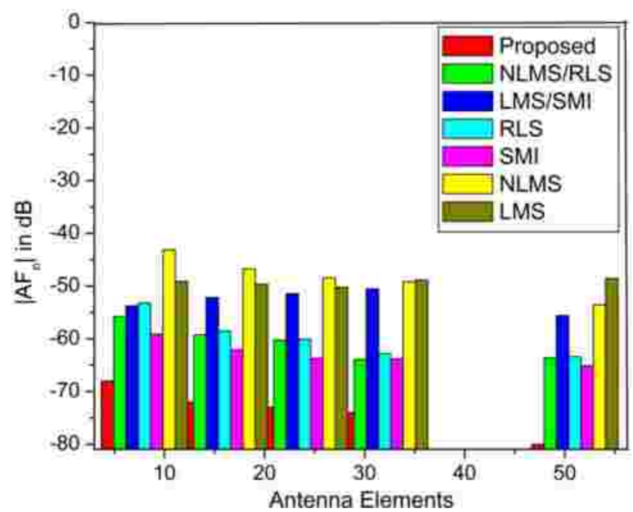

Fig. 11 Average nulls depth in decibels at 0° versus antenna elements

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Nagatsuka M, Ishii N, Kohno R, Imai H (1994) Adaptive array antenna based on spatial spectral estimation using maximum entropy method. IEICE Trans Commun E77-B:624–633
2. Ko CC (1993) A fast adaptive null-steering algorithm based on output power measurements. IEEE Trans Aerosp Electron Syst 29:717–725
3. Ko CC, Balabshaskar G, Bachl R (1993) Unbiased source estimation with an adaptive null steering algorithm. Signal Process 31:283–300
4. Dentino M, McCool J, Widrow B (1978) Adaptive filtering in frequency domain. Proc IEEE 66:1658–1659
5. Narayan SS, Peterson AM (1981) Frequency domain least mean-square algorithm. Proc IEEE 69:124–126
6. Bakhar M, Vani RM (2016) Robust blind beam formers for smart antenna system using window techniques. Elsevier Proc Comput Sci 93:713–720
7. Chen RY, Wang CL (1990) On the optimum step size for the adaptive sign and LMS algorithms. IEEE Trans Circuits Syst 37:836–840
8. Bakhar Md, Vani RM (2014) Implementation and optimization of modified MUSIC algorithm for high resolution DOA estimation. In: Proc. in IEEE int. microwave and RF conf., pp 190–193. https://doi.org/10.1109/imarc.2014.7038985

9. Mathews VJ, Cho SH (1987) Improved convergence analysis of stochastic gradient adaptive filters using the sign algorithm. IEEE Trans Acoust Speech Signal Process 35:450–454
10. Jones MA, Wickert MA (1995) Direct sequence spread spectrum using directionally constrained adaptive beamforming to null interference. IEEE J Select Areas Commun 13:71–79
11. Bakhar Md, Vani RM (2017) Robust beamforming algorithms for smart antenna systems. IUP J Telecommun IX(1):40–53
12. Slock DTM (1993) On the convergence behavior of the LMS and the normalized LMS algorithms. IEEE Trans Signal Process 41:2811–2825
13. Rupp M (1993) The behavior of LMS and NLMS algorithms in the presence of spherically invariant processes. IEEE Trans Signal Process 41:1149–1160
14. Barrett M, Arnott R (1994) Adaptive antennas for mobile communications. Electron Commun Eng J 6:203–214
15. So HC, Chan YT (2003) Analysis of an LMS algorithm for unbiased impulse response estimation. IEEE Trans Signal Process 51(7):1–6
16. Srar A, Kah-Seng C (2009) Adaptive RLMS algorithm for antenna array beamforming. In: IEEE region 10 conference TENCON, pp 1–6
17. Ali RL (2012) A robust least mean square algorithm for adaptive array signal processing. Wirel Pers Commun 68:1449–1461
18. Dungriyal K, Anand S, Kumar S (2014) Performance of MIR-LMS algorithm for adaptive beam forming in smart antenna. In: Proceedings of IRF international conference, Chennai, 20th April 2014
19. Veerendra, Bakhar Md (2017) A novel LMS beamformer for adaptive antenna array. Elsevier Proc Comput Sci 115:94–100. https://doi.org/10.1016/j.procs.2017.09.081
20. Veerendra, Bakhar Md (2018) Efficient blind beamfoming algorithms for phased array and MIMO radar. IETE J Res 64(2):241–246. https://doi.org/10.1080/03772063.2017.1351319
21. Shubair RM, Merri A, Jessmi W (2018) Improved adaptive beamforming using a hybrid LMS/SMI approach. In: Proceedings of IEEE wireless and optical communications networks conference, pp 603–606. https://doi.org/10.1109/wocn.2005.1436097
22. Khalaf AAM, El-Daly ARBM (2018) Hamed HF (2018) A hybrid NLMS/RLS algorithm to enhance the beamforming process of smart antenna systems. Electron Comput Eng 10(1–4):15–22
23. Han FM, Zhang XD (2005) An ESPRIT-like algorithm for coherent DOA estimation. IEEE Antennas Wirel Propag Lett 4:443–446
24. Qiu L, Cai Y, De Lamare RC, Zhao M (2016) Reduced-rank doa estimation algorithms based on alternating low-rank decomposition. IEEE Signal Process Lett 23(5):565–569
25. Godara LC (1997) Application of antenna arrays to mobile communications. Ii. Beam-forming and direction-of-arrival considerations. Proc IEEE 85(8):1195–1245
26. Zhang Y, Poh Ng B (2010) MUSIC-like DOA estimation without estimating the number of sources. IEEE Trans Signal Process 58(3):1668–1676
27. Dakulagi V, Bakhar M (2019) Smart antenna system for DOA estimation using single snapshot. Wirel Pers Commun 107(1):81–93