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Adaptive Beamformers for High-Speed Mobile Communication

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Abstract
In this work, we propose a fast conjugate gradient method (CGM) for beamforming, after thoroughly analyzing the performances of the least mean square (LMS), the recursive least square (RLS), and the sample matrix inversion (SMI) adaptive beamforming algorithms. Various experiments are carried out to analyze the performances of each beamformer in detail. The proposed conjugate gradient method does not use the Eigen spread of the signal correlation matrix as in the case of the LMS and the RLS methods. It computes antenna array weights orthogonally for each iteration. Hence the convergence rate and the null depths of the proposed method are much better than the LMS, the SMI the RLS and the classical CGM. Also, the simulation results confirm that this method has a speed improvement of about 60% over the classical conjugate gradient method. This aspect significantly reduces the processor burden and saves a lot of power during the beamforming process. Hence the proposed method is superior compared to the LMS, the RLS, the SMI, and classical CGM and most suitable for high-speed mobile communication.

Keywords CGM · LMS · RLS · SMI · Smart antenna

1 Introduction

An adaptive beamformer [1–5] shown in Fig. 1 is a novel technology and it has been used in the wireless communication systems for many years. By administering advanced networkability, it enhances the revenues of network operators and provides fewer chances of discarded calls to consumers.

An adaptive beamformer performs spatial signal processing adaptively. It consists of an array of transmitters and receivers. These systems are initially developed in the early 1960s for radar [6, 7] and sonar [8, 9]. The modern applications of adaptive beamformers includes radio telemetry, long term evolution (LTE) [10], imaging [11], seismology [12], mobile sensor networks [13], biomedicine [14], 5G cellular communication [15], IEEE802.16 WiMax [16], W-CDMA [17], and UTM [18].

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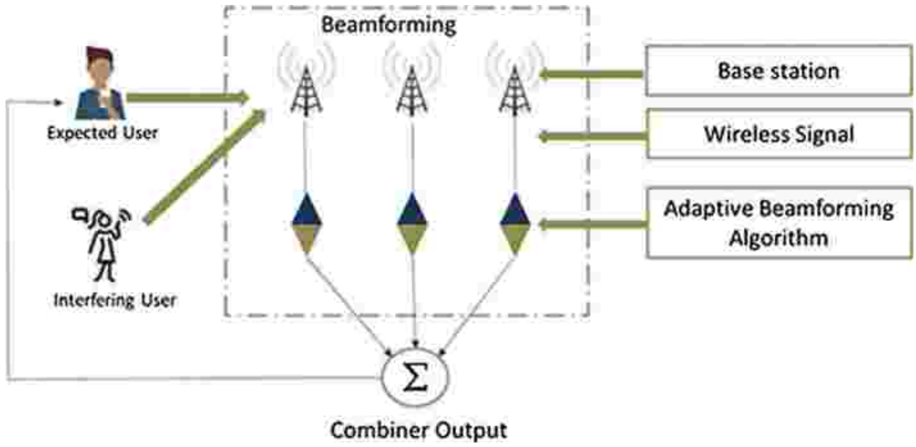


Fig. 1 Block representation of an adaptive beamformer

There are several adaptive beamforming methods are available in the literature for wireless communication, among them, most popular are the LMS, the RLS, the SMI and the CGM algorithms. However, most of these methods present the difficulty of practical implimentation due to many aspects [19, 20]. In the literature, the LMS is the simplest and the most studied approach among all the beamforming methods [21]. This beamforming algorithm requires a large number of iterations for the beamforming which makes it unfit for a few wireless communication applications [22]. To solve this problem, the RLS algorithm uses a gain matrix in place of gradient step size. The RLS algorithm requires less iterations, has better nulling and null depth than the LMS approach [23, 24]. The SMI algorithm is a fast/nulling beamforming approach because of the direct computation of the covariance matrix [25, 26]. It uses the matrix inversion technique which avoids the iterations for the beamforming. This method is much better than the LMS and the RLS in most applications [27–29].

The CGM [30] enhances the convergence speed by using conjugate paths for each new iteration to provide the optimum solution. The CGM technique has fast convergence rate and good null depth than the aforementioned methods [31–33].

In this research, convergence rate and null depths of the proposed beamformer is improved by improving the classical CGM algorithm. The proposed algorithm provides about 60% of improvement in convergence speed over the classical CGM.

2 Array Signal Model

Consider a uniform linear antenna array (ULA) composed of L antenna elements, $L = (1, 2, \ldots, L-1)$ for a DOA problem. Let, M number of source signals are impinging from directions $\left[(\theta_o), (\theta_1), (\theta_2), \ldots, (\theta_{L-1})\right]$. Let the spacing between each antenna elements be $d = 0.5\lambda$, where λ is the wave number of incoming signals. The signal received at the ULA is expresses as

$$\mathbf{x}(n) = \left[\mathbf{a}\left(\theta_o\right), \mathbf{a}\left(\theta_1\right), \mathbf{a}\left(\theta_2\right), \ldots, \mathbf{a}\left(\theta_{L-1}\right)\right]\mathbf{s}(n) + \mathbf{n}(n) = \mathbf{A} \cdot \mathbf{s}(n) + \mathbf{n}(n).$$

Here, $\mathbf{a}\left(\theta_i\right)$=Steering vector of L-element for the θ_i directions. $\mathbf{s}(n)$=Induced signal vector, which is a complex micro chromatic at time n. $\mathbf{n}(n)$=Noise vector at each antenna element, it has zero mean and σ_n^2 variance. $\mathbf{A} = \left[\mathbf{a}\left(\theta_o\right), \mathbf{a}\left(\theta_1\right), \mathbf{a}\left(\theta_2\right), \ldots, \mathbf{a}\left(\theta_{L-1}\right)\right]$, $L \times L$ array manifold vector.

This array manifold vector includes all possible steering vectors.

3 Adaptive Beamforming Algorithms

In adaptive beamforming technique, antenna groups are utilized to focus the maximum power in the direction of the intended signal while discarding matching frequency signals from unwanted directions. Weights of the antenna are computed and updated adaptively using signal samples [19–21].

This adaptive method allows a fine beam in the intended path plus decreased yield in additional paths, which brings effect in noteworthy enhancement in signal-to-interference-plus-noise ratio (SINR). Base stations are exclusively used to transmit each received signal from client by using this technique.

This significantly diminishes the complete intrusion in the system. This is accomplished by varying the weights applied to a single antenna used in the array [22–25].

3.1 The LMS Algorithm

The LMS beamforming approach is quite simple and exploited in many wireless communication applications until today [26]. This method can perform beamforming without the requirement of matrix inversion which is used in the SMI method. It uses a fixed step size for beamforming. This makes the LMS the most competent and simple. Hence this approach is usually most commonly considered adaptive beamforming technique in various applications.

The LMS method can be designed using the following weight equation

$$w(n+1) + w(n) + \mu e^{*}(n)\mathbf{x}(n) \tag{1}$$

Here, $e(n) = \mathbf{s}(n) - w^H(n)\mathbf{x}(n)$ and $\mu = \frac{2}{3tr(\mathbf{R}_{xx})}$

The antenna array output for this method is

$$output\ :\ y(n) = w(n) * \mathbf{x}(n)$$

Here, w(n) = weight vector, μ = step size, e = error signal and $\mathbf{R}_{\text{xx}}$ = autocorrelation matrix.

3.2 The Sample Matrix Inversion (SMI) Algorithm

This beamformer is also known as 'direct matrix inversion' (DMI). The SMI scheme uses K-time snapshots to estimate the average time of ACM. This scheme uses optimum Weiner solution to calculate the antenna array weights and it is given by

$$w(n) = \mathbf{R}_{xx}^{-1}\,\mathbf{r}_{xs} \tag{2}$$

Here, $\mathbf{R}_{xx}^{-1}$ represents the inverse of autocorrelation matrix of $\mathbf{R}_{xx}$. $\mathbf{r}_{xs}$ is the cross-correlation.

The autocorrelation matrix and cross-correlations are expressed as

$$\mathbf{R}_{xx} = \mathrm{E}[\mathrm{X}\,\mathrm{X}^{H}] \quad \text{and} \quad \mathbf{r}_{xs} = \mathrm{E}\left[\mathrm{X}S^{H}\right]$$

where X is the matrix of the induced signal. X^H is the hermitian transpose of the matrix X. S is the matrix of a reference signal and E is the expectation operator.

The correlation matrix $\mathbf{R}_{xx}$ can be estimated using the following expression

$$\mathbf{R}_{xx} = \frac{1}{\mathrm{K}} \sum_{n=1}^{k} \mathbf{x}(n)\mathbf{x}(n)^{\mathrm{H}} \tag{3}$$

where k is the observation vector.

The correlation matrix $\mathbf{r}_{xs}$ can be expressed as

$$\mathbf{r}_{xs} = \frac{1}{k} \sum_{n=1}^{k} \mathbf{s}^{*}(n)\mathbf{x}(n) \tag{4}$$

This method is also known as a block adaptive approach since it uses a block of data and it estimates the antenna array weights by block by block. The array factor for beamforming can be calculated using the expression:

$$AF = \sum_{i=1}^{L} w^{H}(i)\; e^{j\,2\,\pi\,d\;\sin(\theta)} \qquad -90^{\circ} \le \theta + 0.001 \le +90^{\circ} \tag{5}$$

3.3 The Recursive Least Square (RLS) Algorithm

The RLS algorithm is one of the most popular adaptive beamforming algorithms in array signal processing. It has a fast convergence rate as compared to the LMS and the SMI schemes.

An important aspect of the RLS scheme is that this method does not require the inversion of a matrix. Hence this method has a better convergence rate and can also be used for large antenna array-based communication systems. The required correlation vector and the correlation matrix can be calculated recursively. These parameters can be expressed as

$$\mathbf{R}_{xx}(n) = \sum_{n=1}^{k} \mathbf{x}_k(n)\mathbf{x}^{H}(n) \tag{6}$$

$$\mathbf{r}(n) = \sum_{i=1}^{k} \mathbf{s}^{*}(n)\mathbf{x}(n) \tag{7}$$

Here, k is the block length. The weighted estimate of the above expression can be obtained as

$$\begin{aligned}\mathbf{R}_{xx}(n) &= \sum_{n=1}^{k} \xi^{k-1}\mathbf{x}_k(n)\mathbf{x}_k^{H}(n)\\ \mathbf{r}(n) &= \sum_{n=1}^{k} \xi^{k-1}\mathbf{s}^{*}(n)\mathbf{x}_k(n)\end{aligned} \tag{8}$$

Here, ξ is the 'forgetting factor' and it is sometimes also called "*exponential weighting of factor rank (2)*". Here the value of 'ξ' is a positive constant $(0 \le \xi \le 1)$.

$$\mathbf{R}_{xx}(n) = \xi \mathbf{R}_{xx}(k-1) + \mathbf{x}(n)\mathbf{x}^H(n) \tag{9}$$

$$\mathbf{r}(n) = \xi \mathbf{r}(k-1) + \mathbf{s}^*(n)\mathbf{x}(n) \tag{10}$$

4 The Proposed Conjugate Gradient Method

The eigenvalue spread of the array covariance matrix is the problem with the steepest descent schemes. The greater eigenvalue spreads result in slower convergences. The conjugate gradient approach accelerates the convergence rate of the beamforming algorithm. It searches perpendicular (conjugate) paths for every iteration to provide an optimum solution. The proposed CGM approach is an iterative method and its main goal is to reduce the quadratic cost function

$$J(w) = \frac{1}{2} w^H w - s^H w \tag{11}$$

Let us take the gradient of cost function as:

$$\nabla_w J(w) = \vartheta w - \mathbf{s} \tag{12}$$

Where ϑ is the $k \times L$ is array matrix samples. The weight update equation of classical CGM is

$$w(n+1) = w(n) - \mu(n)\mathbf{D}_n(n) \tag{13}$$

Here, the value of step size is given by

$$\mu(n) = \frac{\mathbf{r}^H(n)\vartheta\vartheta^H\mathbf{r}(n)}{\mathbf{D}_n^H(n)\vartheta^H\vartheta\mathbf{D}_n(n)} \tag{14}$$

Here, D_n is the direction vectors r(n) is residual vector respectively.

Now let us update the direction vector and residual vector as:

$$\mathbf{D}_n(n+1) = \vartheta^H\mathbf{r}(n+1) - \alpha(n)\mathbf{D}_n(n) \tag{15}$$

$$\mathbf{r}(n+1) = \mathbf{r}(n) + \mu(n)\vartheta\mathbf{D}_n(n) \tag{16}$$

Now, the value of $\xi(n)$ can be obtained by the use of linear search which minimizes $J(w(n))$.

$$\xi(n) = \frac{\mathbf{r}^H(n+1)\vartheta\vartheta^H\mathbf{r}(n+1)}{\mathbf{r}^H(n)\vartheta\vartheta^H\mathbf{r}(n)} \tag{17}$$

Now, using (13)–(17), the new CGM technique for efficient beamforming is devised as follows.

The correlation matrix **R** can be estimated using the following expression as:

$$\mathbf{R} = \frac{1}{k} \sum_{n=1}^{k-1} \mathbf{x}(n-k)\mathbf{x}^H(n-k) \tag{18}$$

where k is the observation vector. The correlation matrix $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \frac{1}{k} \sum_{n=1}^{k-1} \mathbf{s}^*(n-k)\mathbf{x}(n-k) \tag{19}$$

Now the weight of the proposed CGM method is expressed as

$$w_k(n) = w_k(n-1) - \alpha_k(n)\ell_{k-1}(n) \tag{20}$$

where $w_k(n)$ is the kth weight vector at time n. $\alpha_k(n)$ is the step size which is required to update the weights. $\ell_{k-1}(n)$ is the parameter for searching the direction. The expression of this parameter can be obtained as

$$\ell_k(n) = f_k(n) + \hbar_k(n)\ell_{k-1}(n). \tag{21}$$

Here $f_k(n)$ is the opposite gradient direction of a cost function which is expressed as

$$f_k(n) = -\nabla J\left(w_k(n)\right). \tag{22}$$

The iteration expression of the above value can be written as

$$f_k(n) = f_{k-1}(n) - \alpha_k(n)\mathbf{R}_k(n)\ell_{m-1}(n) \tag{23}$$

The value of the $a_k(n)$ is given in the following lines to have the minimum cost function as

$$\alpha_k(n) = -\left[\frac{\ell_{k-1}^H(n)f_{k-1}(n)}{\ell_{k-1}^H(n)\mathbf{R}_k(n)\ell_{k-1}(n)}\right] \tag{24}$$

The parameter $\hbar_k(n)$ mentioned in (21) is very useful to obtain the $\mathbf{R}_k(n)$ orthogonally for the direction vector $\left\{\ell_k(n), (k = 1, 2, \ldots)\right\}$. The expression of $\hbar_k(n)$ can be expressed as

$$\hbar_k(n) = -\left[\frac{f_k^H(n)\mathbf{R}_k(n)\ell_{k-1}(n)}{\ell_{k-1}^H(n)\mathbf{R}_k(n)\ell_{k-1}(n)}\right] \tag{25}$$

The expressions from (18)–(25) represent the process of the proposed CGM algorithm. Finally, the array factor for beamforming can be calculated as

$$AF = \sum_{i=1}^{L} w^H(i)\ e^{j2\pi d\ \sin(\theta)} \qquad -90^\circ \le \theta + 0.001 \le +90^\circ \tag{26}$$

5 Results and Discussion

Computer simulations have been conducted for various adaptive beamforming algorithms to study the performance of well-known methods and the proposed method. In these simulations, L element arrays with inter- element spacing $\mathrm{d} = 0.5\lambda$ are considered. The variance of $\sigma^2 = 0.01$ is assumed. In each method, the array element weights are calculated according to the design equations of the adaptive beamformer. Also, array factors are calculated and plotted for the range $-90^{o} \le \theta \le 90^{o}$.

5.1 Analysis of the LMS Algorithm

Now, firstly let us study the performance of the LMS beamformer in detail. The simulation parameters used in this method are tabulated in Table 1.

The simulated results of the LMS algorithm are shown below. Figure 2a and b are the radiation pattern in rectangular and polar form respectively. The normalized array pattern of the LMS algorithm is shown in Fig. 2c. The LMS algorithm is the first choice of communication engineer when a simple beamforming method is required. Though it less complex, requires huge iterations (about 80) before producing the signal. Hence this method is not recommended method fast communication applications.

5.2 Analysis of the SMI Algorithm

The simulation parameters used for SMI scheme are as follows:

Consider a ULA array configuration with array elements = 10; array antenna spacing (d) = λ/2; angle of desired signal = $\theta_0 = 0°$, angle of interference signal $\theta_1 = -60°$. Let block length = 30. Consider a white Gaussian noise with variance $\sigma_n^2 = 0.01$. The AF plots in rectangular and polar forms are shown in Fig. 3a and b respectively. The normalized array pattern of the SMI algorithm is shown in Fig. 3c.

The SMI method uses computation of the inverse of the autocorrelation matrix which leads to error. Hence this method is not suitable for communication systems in which large antenna arrays are required.

5.3 Analysis of the RLS Algorithm

Let us consider a number of elements = 06, a number of snapshots = 200, the spacing between the array elements, $d = \lambda/2$, the value of $\xi = 0.91$, AOA of the interference signal = −60° and AOA of induced signal = 0°. The radiation pattern in rectangular and polar forms is shown in Fig. 4a, b respectively. The normalized array pattern of the RLS algorithm is shown in Fig. 4c.

Table 1 Simulation parameters of the LMS algorithm

S. no	Parameters	Values
1	Type of antenna configuration	ULA
2	Antenna elements	8
3	Inter element spacing	$d = 0.5\lambda$
4	AOA of induced signal	0^0
5	Interferences	[−20° 30°]
6	Values of step size	$\mu = 0.2$

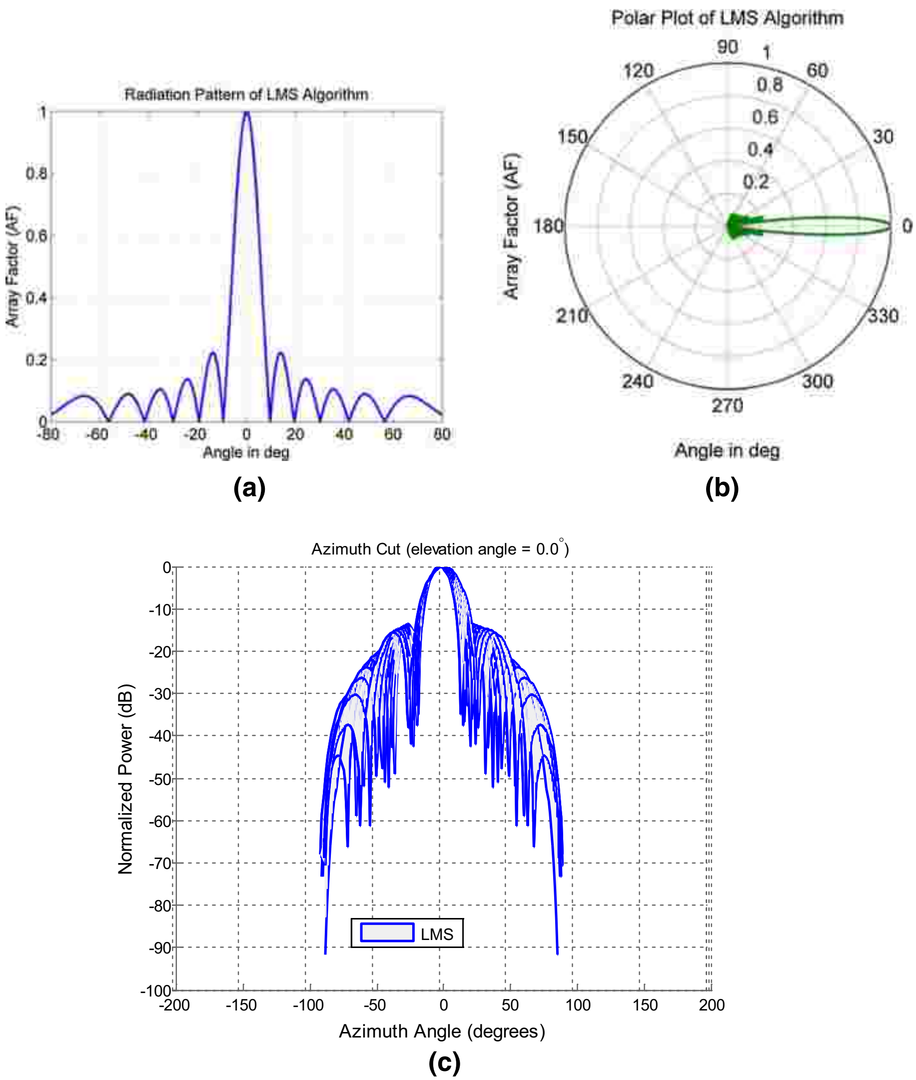


Fig. 2 **a** Radiation pattern. **b** Polar pattern. **c** Normalized array pattern, L = 8

Figure 5a, b show the convergence of weight vector and convergence of estimated parameters of the RLS algorithm respectively.

It can be noticed that the error of this algorithm decreases for each iteration.

5.4 Results and Analysis of the Proposed Algorithm

The simulation parameters used for the implementation of the proposed CGM algorithm are as follows,

Consider the following parameters used for the simulation of the proposed algorithm. Let inter-element spacing (d) = $\lambda/2$, DoA = $\theta_0 = 0°$, angle of interference = $\theta_1 = -30°$,

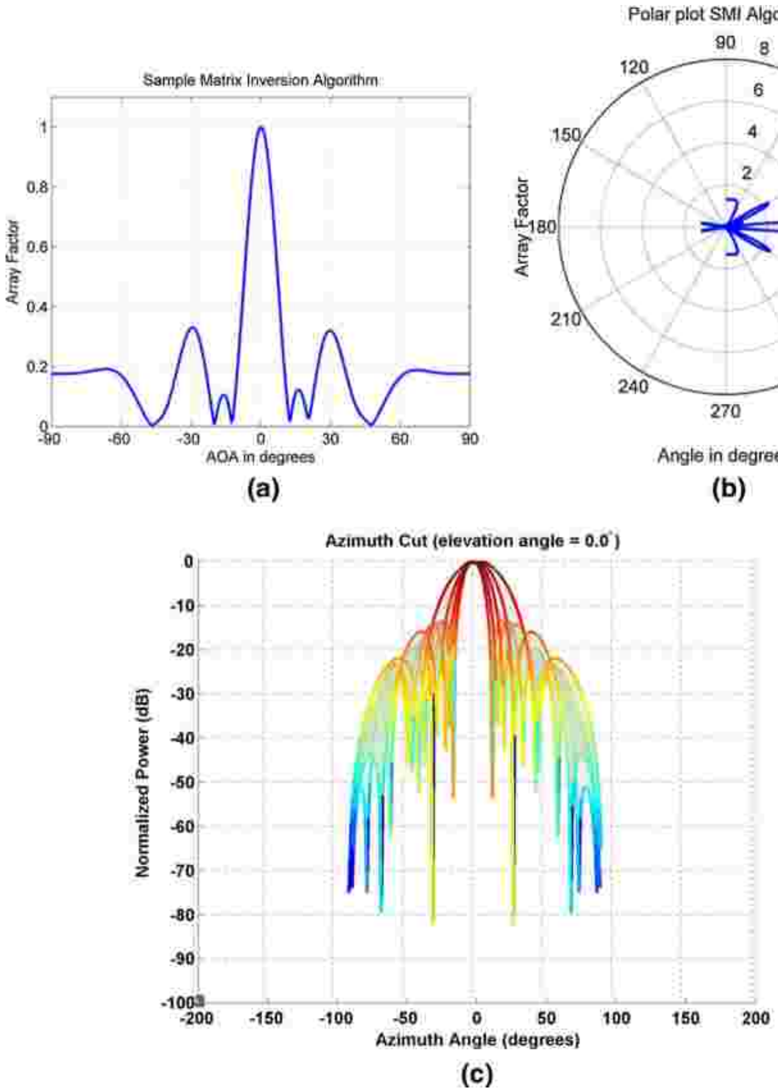


Fig. 3 **a** Radiation pattern. **b** Polar pattern. **c** Normalized array pattern L = 5

number of samples = 200, forgetting factor $\xi = 0.91$, number of array elements = 5 and 40 and additive white noise with variance $\sigma_n^2 = 0.01$. The results obtained for the less array elements (N = 5) are shown below. Figure 6a and b represent the rectangular and polar forms of radiation patterns obtained for the proposed method for L = 5 antenna elements. The real part of the desired signal at the array output is shown in Fig. 7.

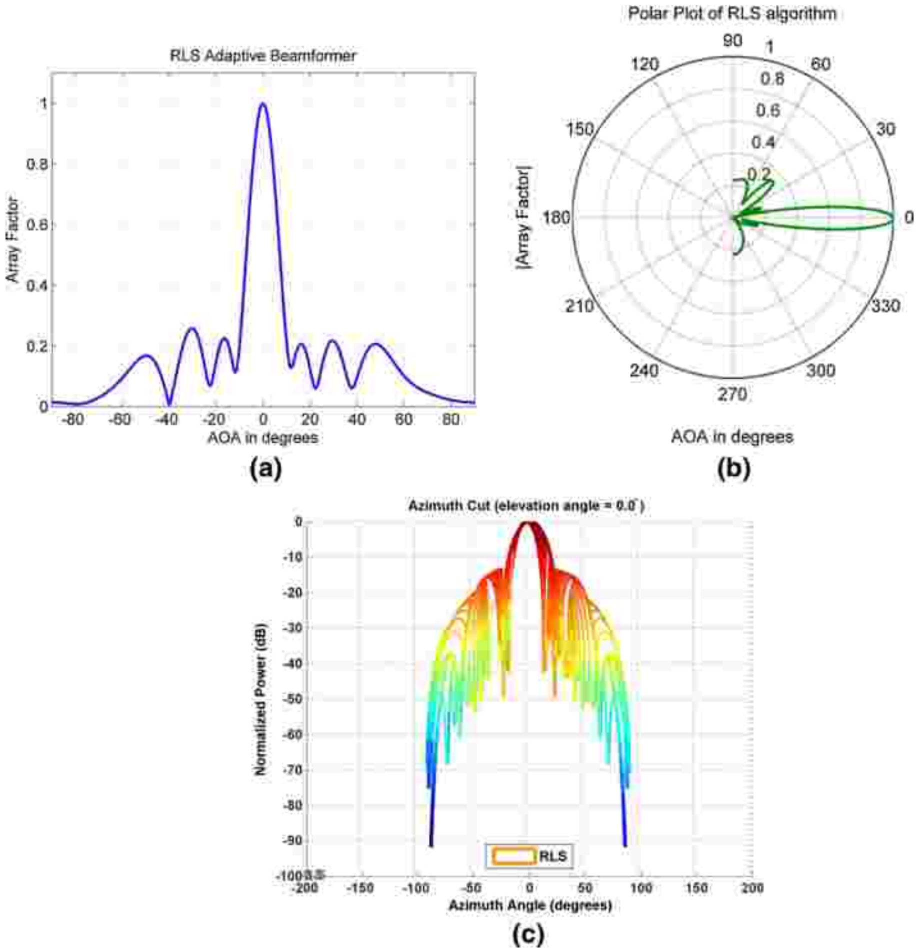


Fig. 4 **a** Radiation pattern. **b** Polar pattern. **c** Normalized array pattern, L = 5

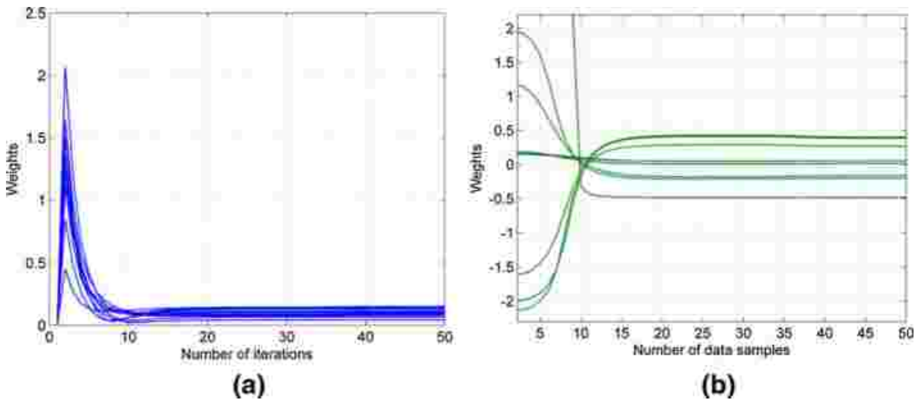


Fig. 5 **a** Convergence of weight vector. **b** Convergence of estimated parameters

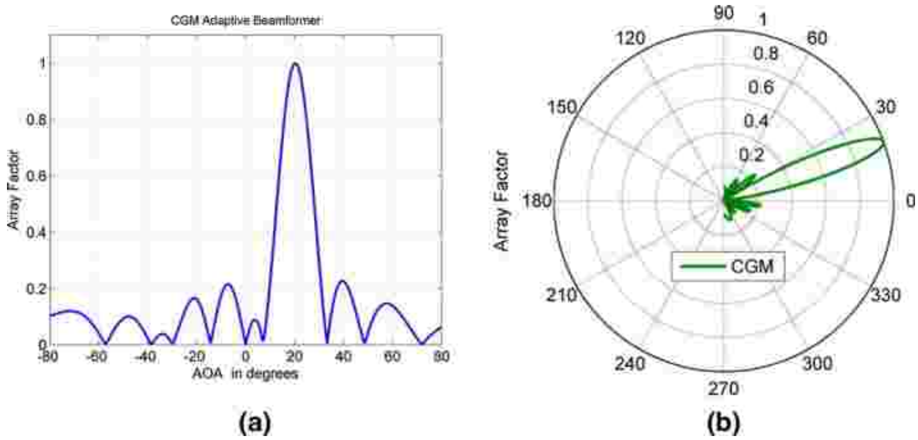


Fig. 6 Radiation pattern: **a** rectangular pattern. **b** Polar pattern for less array elements

The normalized antenna array pattern is shown in Fig. 8. The normal and the three-dimensional (3D) view of the normalized antenna array pattern is shown in Fig. 8a, b respectively.

The proposed method can be applied for the larger array elements. When the array elements L = 4, the antenna weights are as follows. The simulation results for this case are shown from Figs. 9, 10 and 11.

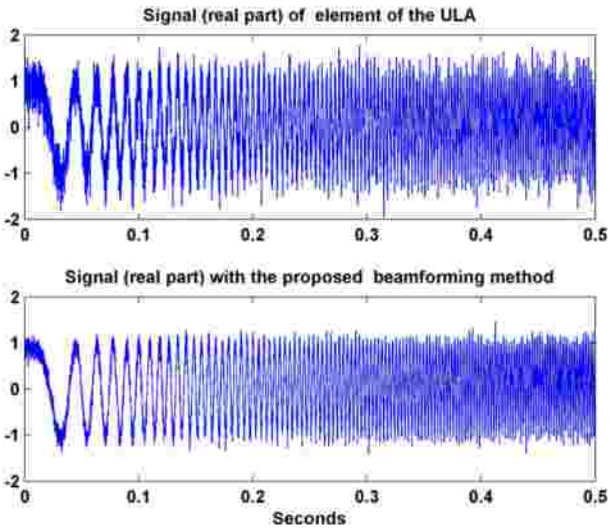


Fig. 7 The real part of the proposed algorithm

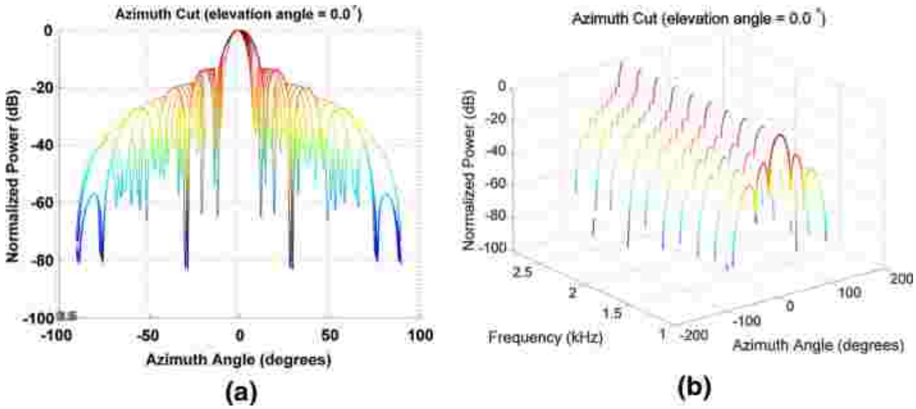
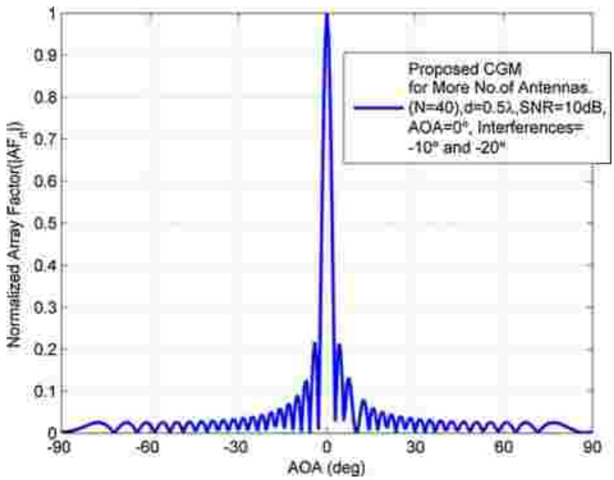


Fig. 8 Normalized power Pattern of the proposed method **a** normal view, **b** 3D-view for less array elements

Fig. 9 Normalized array pattern for large array elements (L = 40)



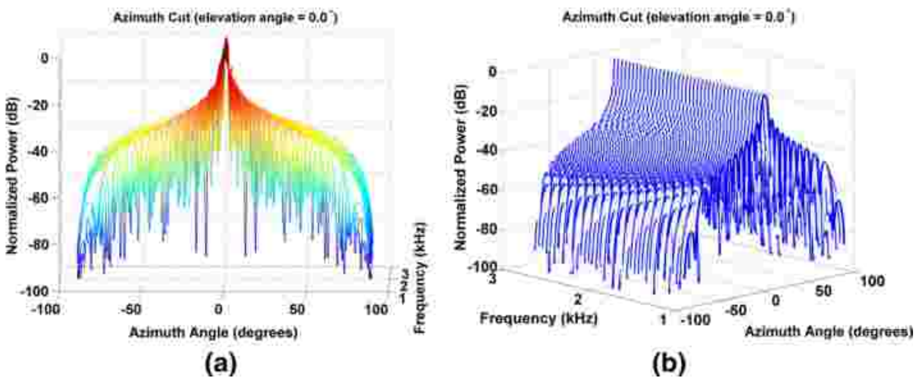


Fig. 10 Normalized power pattern of the proposed method **a** normal view, **b** 3D-view for less array elements

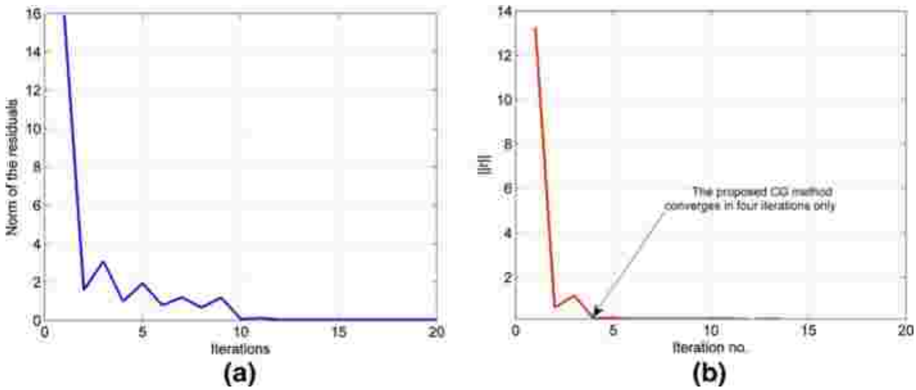


Fig. 11 Plot of convergence **a** classical CGM. **b** The proposed CGM

w1 = 1, w2 = 0.95633 − 0.026015i, w3 = 0.93249 − 0.070915i, w4 = 0.93541 − 0.12167i w5 = 0.96424 − 0.16353i, w6 = 1.0106 − 0.18437i, w7 = 1.0611 − 0.17812i, w8 = 1.1009 − 0.1466i, w9 = 1.1187 − 0.098956i, w10 = 1.1091 − 0.049027i, w11 = 1.0751 − 0.011303i, w12 = 1.0263 + 0.0032649i, w13 = 0.97716 − 0.0095522i, w14 = 0.94175 − 0.046034i, w15 = 0.93042 − 0.095589i, w16 = 0.94644 − 0.14383i, w17 = 0.98517 − 0.17676i, w18 = 1.0354 − 0.18481i, w19 = 1.0825 − 0.16565i, w20 = 1.1128 − 0.12484i, w21 = 1.1175 − 0.074223i, w22 = 1.0953 − 0.0285i w23 = 1.0526 − 0.00094008i, w24 = 1.0017 + 0.00045527i, w25 = 0.95758 − 0.024719i, w26 = 0.93289 − 0.069154i, w27 = 0.93484 − 0.11995i, w28 = 0.96286 − 0.16236i, w29 = 1.0088 − 0.18408i, w30 = 1.0594 − 0.1788i, w31 = 1.0999 − 0.14804i, w32 = 1.1185 − 0.10075i, w33 = 1.1099 − 0.050651i, w34 = 1.0766 − 0.012282i, w35 = 1.0282 + 0.0032153i, w36 = 0.97872 − 0.008658i, w37 = 0.94263 − 0.044455i, w38 = 0.93035 − 0.093784i, w39 = 0.94545 − 0.14233i, w40 = 0.98354 − 0.17599i.

The normalized radiation pattern of the proposed method for large array elements is shown in Fig. 9. The normal and three-dimensional normalized array pattern is shown in Fig. 10a and b respectively. The convergence analysis of the classical CGM and the proposed method is shown in Fig. 11a and b respectively. From the above figures we note that the proposed conjugate gradient method has the fastest convergence rate over all the methods discussed in the paper. The improvement factor of the proposed method over classical CGM is calculated as follows.

5.5 Calculation of Improvement Factor

Let us consider the number of iterations of the CGM algorithm for beamforming $= \xi_L = 10$.

Also, the number of iterations of the proposed method for beamforming $= \xi_N = 4$

Table 2 Comparison of adaptive beamforming algorithms

Algorithm	Iteration required for beamforming	Performance of beamformer
LMS	80	Low convergence rate and not suitable for high-speed wireless applications [1–4, 6]
RLS	15	Less efficient in a non-stationary environment
SMI	Block by block (no iteration)	It requires large computations and not suitable for large antenna array [7, 8]
CGM	10	The CGM method is one of the fastest beamforming methods. This method is more complex than the LMS method [30]
CMA	More than 100	The CMA method is the blind beamforming technique used in array signal processing. However, it's very low convergence speed limits its use in many wireless applications. It needs significant improvement [6, 13]
LS-CMA	16	This algorithm requires about 16 iterations only. Hence LS-CMA is much better than the CMA and more immune to noise [6, 13]
NLMS	60	This method has better nulling/beamforming than the LMS [30–33]
VSSLMS	50	This method has better nulling/beamforming than the LMS, and the NLMS [30–33]
VSSNLMS	50	Unlike, the LMS, the NLMS, this approach uses the variable step size which improves the convergence rate
RLMS	20	Robust LMS algorithm is much better than the LMS, NLMS and VSSNLMS [3–6]
LMS–LMS	25	Convergence is lower than RLS,CGM,SMI, RLMS [19]
LMS with SMI	22	It requires large computations not suitable for high-speed wireless applications
The proposed	4	Very fast (convergence speed) as compared to any beamforming methods. And most suitable for high-speed wireless communication applications

$$
\begin{aligned}
\textit{Improvement factor} &= \left(\frac{\xi_{\mathrm{L}} - \xi_{\mathrm{N}}}{\xi_{\mathrm{L}}}\right) \\
&= \left(\frac{\xi_{\mathrm{L}} - \xi_{\mathrm{N}}}{\xi_{\mathrm{L}}}\right) \times 100 \ \ \textit{in percentage} \\
&= \left(\frac{10 - 4}{10}\right) \times 100 \\
&= 60\%.
\end{aligned}
$$

The above calculation and the experimental results illustrate that the proposed method has about 60% of improvement over the classical CGM algorithm. This significantly reduces the processor speed and saves a lot of power. Hence this method is most suitable for 5G and beyond cellular communication. A comparison of various popular beamformers is tabulated in Table 2.

6 Conclusion

The main concern of this research is to analyze various beamforming methods and to propose a fast beamforming method. The LMS is the most popular and studied beamforming method among the adaptive beam formring algorithms. But the key demerit of this technique is that it requires a large number of iterations before beamforming. It takes more iterations to produce the required beam which is an undesirable and inefficient method and not suitable for high-speed mobile communication.

Computer simulations are evident that the proposed CGM beamforming method exhibits superior performance in terms of convergence rate, accuracy, null depths, speed and robustness as compared to the LMS, the RLS, the SMI and classical CGM methods. The proposed method has about 60% of improvement over the classical CGM method. Hence the communication system using this method can provide enhanced system capacity as compared to the other methods.

Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

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