HKE Society's

B V Bhoomaraddi College of Arts, Science and Commerce, Bidar

Affiliated to Gulbarga University, Kalaburagi, Approved by UGC under 2(f) &12(B)
Accredited by NAAC –"A" Grade in 3rd Cycle with CGPA 3.07, ISO 9001:2015 Certified

CERTIFICATE

KSTA Sponsored

International Conference on Physics and Allied Sciences (ICPAS-2020)

11th - 13th March, 2020

This is to certify that

Dr./Prof./Mr./Miss/Mrs. Dr. Veerendra Dakulagi

From Dept.of E&CE, Lincoln University College, PJ, Malaysia

Has presented a paper entitled

A New Coherent DOA Estimator for Radar Communication

in the KSTA Sponsored International Conference on Physics and Allied Sciences (ICPAS) organized by Department of Physics, B V Bhoomaraddi College of Arts, Science and Commerce, Bidar in collaboration with Karnataka Science and Technology Academy (KSTA), Department of IT, BT and Science and Technology, Government of Karnataka (GoK)

Dr. A M Ramesh
CEO, KSTA, Bengaluru

Dr. S K Satnoor
Principal & General Chair

A New Coherent DOA Estimator for Radar Communication

Veerendra Dakulagi and Mukil Alagirisamy
Department of E&CE, Lincoln University College, 47301 Petaling Jaya, Malaysia

---

Abstract

Nystrom based direction of arrival (DOA) technique to reduce the computational load is proposed in this paper. This algorithm computes singular vector decomposition (SVD) of intended signals without computing sample covariance matrix (SCM). For a uniform linear array (ULA) composed of '*L*' antenna elements, proposed Nystrom based ESPRIT method requires only $O(LM^2+LM)$ flops to compute SVD of SCM when *'M'* number of signals impinge ULA. This makes algorithm computationally efficient especially when the antenna elements or number of snapshots are too large. Computer simulations are carried out and comparison with recent DOA methods are made to demonstrate the effectiveness of the Nystrom based ESPRIT method in wireless communication applications.

Key Words: ESPRIT, Direction-of-arrival, Nystrom, MUSIC, Symmetric ULA.

1.Introduction

We know that array signal processing is one of the rapidly growing areas of communication engineering. In wireless communication, the beam forming and effective direction of arrival estimation using smart antenna systems are more efficient in terms of signal quality[1]-[3]. Multiple signal classification (MUSIC)[4], root MUSIC[5] and estimation of signal parameter using rotational invariance technique (ESPRIT)[6] are the most promising methods for estimating the DOAs of intended signals using multiple antennas. In [7], a reduced complexity beamforming algorithm for mobile commutations is presented. Interference reduction is the key factor to effectively communicate with mobile users. In [8] the performance enhanced beamforming algorithms for multiple-input and multiple-output (MIMO) system is presented. The increased demands for capacity and quality for MIMO operates without bothering for increase in radio frequency spectrum allocation. This factor motivates for new algorithms to improve the need for spectrum utilization. Performance evaluation of handover in long term evolution (LTE) based on power budget handover algorithm is discussed in[9].

The ESPRIT is one of the widely used and perhaps the most studied DOA algorithm in its class [10]-[16]. However, this technique deteriorates from its performance for small sample size and low signal to noise ratio (SNR) conditions. It requires huge computations for large antenna arrays and samples to compute SCM and eigen vector decomposition (EVD). Literature shows few methods used to reduce the complexity of DoA algorithms [17]-[26]. Few popular methods are: DOA estimation via pseudorandom re-sampling of spatial spectrum [27], *lp*-MUSIC [28], M-MUSIC [29], F-MUSIC [30], Unitary root-MUSIC[31] and Nysrom-MUSIC[32].

Speed of the DOA methods can be enhanced by generating the low rank approximations [33]. This can be achieved by the use of Nyström criteria to the subspace methods. Reducing the complexity of the DOA algorithms and making them more robust and suitable for the practical applications like, radar, sonar and wireless communications is the key motivation. Hence, in this work, we used the Nystrom approximation to the popular ESPRIT algorithm to make it a suitable candidate for wireless communications, particularly when the array elements or sample size is more.

2.Array Signal Model

Consider a geometry of ULA with $L = 2N + 1$ antenna elements distributed horizontally with $M(M<L)$ number of source signals as shown in Figure 1. Let the spacing between each antenna element be $d = \lambda / 2$. Here $\lambda = v / f$, where *v* is the speed of light and f is the frequency of received signals respectively. Let received $\vec{x}(n)$ impinges ULA with directions $\theta(\theta_1, \theta_2, ..., \theta_m)$ in far field is $\vec{x}(n) =$ $[x_{-N}(n), ... x_0(n), ... x_N(n)]^T$.

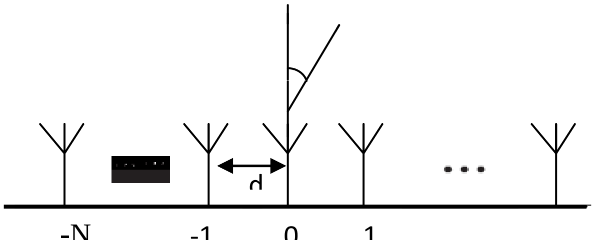


Figure1. Considered Symmetric ULA Model.

The induced signal in noisy environment at n^{th} time can be expressed as

$$\vec{x}(n) = \sum_{i=1}^{M} \vec{a}(\theta_i) * \vec{s}(n) + \vec{n}_o(n) \, , \qquad (1)$$

Here,

$\vec{s}(n)$ = Desired signal

$\vec{a}(\theta_i)$= steering vector of i^{th} interference signal

$\vec{n}(n)$ = White Gaussian noise with zero mean.

The above equation can be expressed in matrix form as

$$\vec{x}(n) = \vec{A}(\theta)\vec{s}(n) + \vec{n}(n) \qquad n = 0,1\cdots N-1 \quad (2)$$

Where

$$\vec{A}(\theta) = \left[\vec{a}(\theta_1), \vec{a}(\theta_2), \cdots, \vec{a}(\theta_M)\right] \qquad (3)$$

$$\vec{s}(n) = \left[\vec{s}_1(n), \vec{s}_2(n), \cdots, \vec{s}_M(n)\right] \qquad (4)$$

are the steering matrix and complex signal vectors respectively.

The SCM required for the DoA estimation can be calculated as

$$\Re = \mathrm{E}\left[\mathrm{X}\mathrm{X}^H\right] \qquad (5)$$

Where

X = [**x**(1), **x**(2),…,**x**(N)] is a data matrix.

3.Proposed Nyström Based DoA Algorithm

In this work, the complexity of MUSIC algorithm is reduced by using Nyström theory. DOA estimation using subspace based method uses intensive computations to calculate singular vector decomposition (SVD). Nystrom is the well known mathematician; he has shown that the SVD of sample covariance matrix (SCM) can be computed without actually computing SCM[2]. We have used this classical method in ESPRIT algorithm to reduce its complexity.

Hence the SCM $\Re = \mathrm{E}\left\{XX^H\right\}$ can be expressed as

$$\Re = \begin{bmatrix} \Re_{11} & \Re_{12} \\ \Re_{12}^{\mathrm{H}} & \Re_{22} \end{bmatrix} \qquad (6)$$

Let the received signal matrix‘ X ’ can be portioned as

$$\mathrm{X} = \begin{bmatrix} \mathrm{X}_1 \\ \mathrm{X}_2 \end{bmatrix} \qquad (7)$$

Here, $\mathrm{X}_1 \in C^{u\times n}$ and $\mathrm{X}_2 \in C^{(u-n)\times n}$ are data sub matrices obtained from the 1st ‘u’ antenna array elements & (L-u) antenna array elements respectively. Here ‘u’ is the user defined signal parameter which satisfies $\mathrm{u} \in (1,2,\ldots \mathrm{L})$. Let us define $\Re_{11} = \mathrm{E}\left[\mathrm{X}_1\mathrm{X}_1^H\right]$, $\Re_{12} = \mathrm{E}\left[\mathrm{X}_1\mathrm{X}_2^H\right]$

$$\Re_{22} = \mathrm{E}\left[\mathrm{X}_2\mathrm{X}_2^H\right]. \qquad (8)$$

The main aim of this work is to approximate the eigenvectors and eigenvalues using low complexity techniques. Now, let us consider $\Re_{11}$ is the nonsingular matrix with rank is ‘M’. Let $\mathbf{D_e} = \begin{bmatrix} \mathrm{X}_{11} \\ \mathrm{X}^H{}_{12} \end{bmatrix} \mathrm{X}_{11}^{-1/2}$ and the eigen value decomposition of matrix $\mathbf{D_e}^{\mathrm{H}}\,\mathbf{D_e}$ and it is $R_D \Lambda_D R_D{}^H$ Let $F = \Lambda_D^{1/2} R_D^H R_D \Lambda_D^{1/2}$ and eigen value decomposition of F is $R_F \Lambda_F R_F{}^H$. Hence the signal subspace $R_S \in C^{m\times n}$ is $R_S = \mathbf{D_e}\, R_D \Lambda_D^{-1/2} R_F$

Now, from the above expressions we can represent the Nyström covariance estimator as

$$\begin{aligned} \chi_{NCE} &= R_S \Lambda_D R_S^H \\ &= \begin{bmatrix} \mathrm{X}_{11} & \mathrm{X}_{12} \\ \mathrm{X}_{12}^H & \mathrm{X}_{12}^H \mathrm{X}_{11}^{-1} \mathrm{X}_{12} \end{bmatrix} \end{aligned} \qquad (9)$$

Hence this method requires only $\Re_{11}$ and $\Re_{12}$ to calculation for DoA estimation.

4.Results and Discussions

A Nystrom based ESPRIT DOA algorithm for spectrum estimation of desired signals is implemented in this section. The parameters used for the simulation are tabulated in Table 1. Let us assume that noise is white Gaussian with σ^2 variance and zero mean. The performance of the proposed method is validated and compared with other popular techniques by means of root mean square error (RMSE) using Monte Carlo simulations.

TABLE 1
Parameters used for the simulation of the proposed algorithm

S.No	Parameters	Values
1	Array Configuration	ULA
2	No. of Antennas (L)	10-70
3	Inter element spacing	λ/2
4	Snapshots	100-800
5	Monte Carlo trials	500
6	SNR	-40dB to 10dB

The Nystrom based DOA algorithm is simulated and performance of the smart antenna system is studied using RMSE and the results are compared with classical MUSIC, root MUSIC, modified MUSIC and unitary MUSIC. These results are shown in Figure 1 form small and largeantenna arrays. The root mean square error (RMSE) for analyzing the performance of various DOA methods is expressed as[6].

$$RMSE = \sqrt{\frac{1}{T}\sum_{M=1}^{T}\left|\hat{\theta}_M - \theta_M\right|^2} \qquad (10)$$

Here, T is the number of Monte Carlo trials.

$\hat{\theta}_M$ is the actual direction of source signal

θ_M is the estimated direction of source signal.

Let us consider two narrowband signals with directions 10^o and 60^o impinge a ULA composed of L=20. Let, snapshots (K)=800, inter-element spacing d=0.5λ with SNR varying from -40dB to 10dB. Histogram for the DoA estimation for the sources arriving at 10^o and 60^o is shown in Figure 2.

The RMSE angle performance for small ULA(by considering L=10) and large ULA (by considering L=40) is shown in Figure 3(a) and (b) respectively. Figure 4 shows the RMSE angle performance when K=100 and SNR is varying from -8dB to 10dB keeping other parameters same.

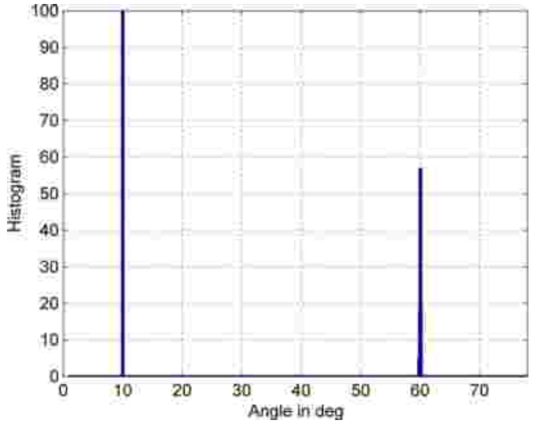
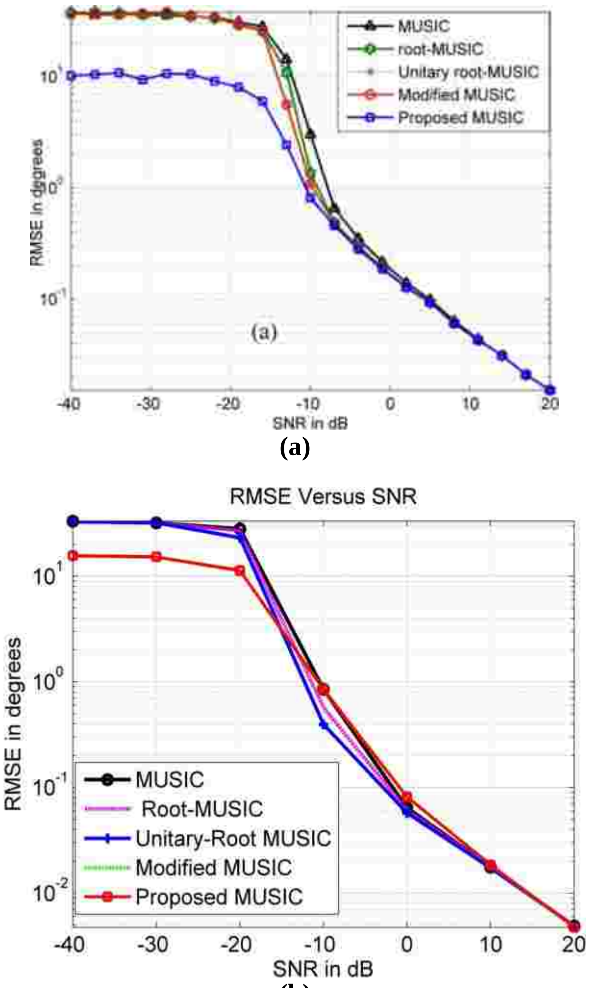


Figure 2: Histogram plot of proposed ESPRIT for 10^o and 60^o targets.



(a)

(b)

Figure 3: RMSE versus SNR for (a) small antenna array(L= 10) (b) Large antenna array(L= 40).

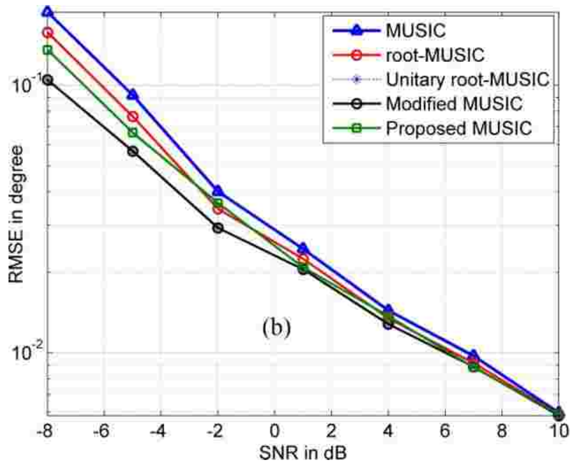


Figure 4: RMSE versus SNR for (L =10, M = 2, K = 100, SNR = -8dB to 10dB).

The time complexity of the proposed method with other popular DoA methods is shown in Figure 5. We used Intel i5-3110M CPU system with 2.40GHz capacity.

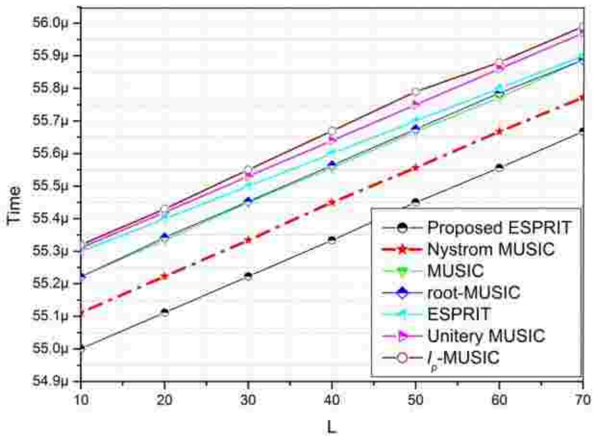


Figure 4: Time calculation of DOA algorithms(in seconds).

From Figures 3 and 4, we note that the proposed and the classical MUSIC algorithm have almost similar RMSE performance. Hence the complexity of MUSIC algorithm is reduced significantly without affecting its performance.

From Figure 5, it is clear that the proposed Nyström based algorithm requires less time as compared to other methods.

4.1 Computational Complexity

The conventional MUSIC uses $O\left(L^3\right)+O\left(L^2K\right)$ flops to compute singular vector decomposition of sample covariance matrix. The proposed Nyström based ESPRIT algorithm calculates SVD of SCM without actually computing SCM. It requires only $O\left(LM^2+LM\right)$ flops, provided that $M < L.$ This makes it computationally efficient and economic.

5.Conclusion

DOA estimation using subspace based methods use intensive computations to calculate SVD. These methods require EVD and sample covariance matrix decomposition to find the signal and noise subspace. The computations will be huge when the number of antenna elements and snapshots increases. Hence computationally efficient algorithms are required for the practical applications. Nystrom theory is a classical algebra which can be used to speed up the algorithms. This new method computes the SVD without computing the sample covariance matrix. Classical MUSIC require $O(L^3) + O(L^2K)$ flops to compute SVD of SCM whereas the Nystrom –DOA algorithm requires only $O(LM^2+LM)$ flops. This makes the proposed algorithm efficient especially when the antenna elements or snapshots are too large. Experimental results presented validate its effectiveness.

Compliance with ethical standards

Conflict of interest: The authors declare that they have no conflict of interest.

References

[1] S. Durrani and M. E. Bialkowski, “Effect of mutual coupling on the interference rejection capabilities of linear and circular arrays in CDMA systems,” IEEE Trans. Antennas Propag., vol. 52, no. 4, pp. 1130–1134, Apr. 2004.

[2] L. C. Godara, “Application of antenna arrays to mobile communications. Ii. Beam-forming and direction-of-arrival considerations,” Proc. IEEE, vol. 85, no. 8, pp. 1195–1245, Aug. 1997.

[3] Veerendra and Md.Bakhar, “Smart Antenna System for DOA Estimation using Single Snapshot”, Wireless Personal Communications, vol. 107, no. 1, pp. 81-93, 2019.

[4] R. Schmidt, “Multiple emitter location and signal parameter estimation,” IEEE Trans. Antennas Propag., vol. 34, no. 3, pp. 276–280, Mar. 1986.

[5] V.-K. Mai, D. Pastor, A. Aïssa-El-Bey, R. Le-Bidan, "Robust estimation of non-stationary noise power spectrum for speech enhancement", *IEEE/ACM Trans. Audio Speech Lang. Process.*, vol. 23, no. 4, pp. 670-682, Apr. 2015.

[6] R. Roy and T. Kailath, “ESPRIT-estimation of signal parameters via rotational invariance techniques,” IEEE Trans. Acoust., Speech, Signal Process., vol. 37, no. 7, pp. 984–995, Jul. 1989.

[7]Zhao Tan, Arye Nehorai, "Sparse direction of arrival estimation using co-prime arrays with off-grid targets", *IEEE Signal Processing Letters*, vol. 21, no. 1, pp. 26-29, 2014.

[8] I Bilik, "Spatial compressive sensing for direction-of-arrival estimation of multiple sources using dynamic sensor arrays", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 3, pp. 1754-1769, 2011.

[9] L. Qiu, Y. Cai, R. C. De Lamare, M. Zhao, "Reduced-rank doa estimation algorithms based on alternating low-rank decomposition", *IEEE Signal Processing Letters*, vol. 23, no. 5, pp. 565-569, 2016.

[10] W.-Q. Wang, "Overview of frequency diverse array in radar and navigation applications", *IET Radar Sonar Navig.*, vol. 10, pp. 1001-1012, 2016.

[11] Bekkerman, J.Tabrikian,Targetdetection and localizationusing MIMO radars and sonars, IEEE Trans. Signal Process, vol.54, no.10, pp3873–3883, 2006.

[12] J. Steinwandt, F. Roemer and M. Haardt, “Generalized Least Squares for ESPRIT-Type Direction of Arrival Estimation,” *IEEE Signal Process. Letters*, vol. 24, no. 11, pp. 1681-1685, 2017.

[13] G. Zheng and B. Chen, “Unitary dual-resolution ESPRIT for joint DOD and DOA estimation in bistatic MIMO radar,” Multidimensional Systems and Signal Process., vol. 26, no. 1, pp. 159-178, 2015.

[14] Kim Juy, J Lee, "Design and implementation of a full-digital pulse-doppler radar system for automotive applications", Cons Elec (ICCE) 2011, pp. 563-564.

[15] Veerendra, Md.Bakhar and Vani.R.M, “Robust Blind Beam formers for Smart Antenna System using Window Techniques”, Elsevier Procedia Computer Science, vol. 93, pp. 713 – 720, 2016.

[16] A.-A. Saucan, T. Chonavel, C. Sintes, and J.-M. Le Caillec, “Cphd-doa tracking of multiple extended sonar targets in impulsive environments,” IEEE Trans. Signal Process., vol. 64, no. 5, pp. 1147–1160, 2016.

[17] Ying Liua,b, Hongyuan Cuia, “Antenna Array Signal Direction of Arrival Estimation on Digital Signal Processor (DSP)”, Procedia Computer Science, Vol. 55 , pp. 782 – 791, 2015.

[18] Lotfi Osman, Imen Sfar, “The application of high-resolution methods for DOA estimation using a linear antenna array”, International Journal of Microwave and Wireless Technologies, vol. 7, no. 1, pp. 87-94, 2015.

[19] B. Porat and B. Friedlander, “Analysis of the asymptotic relative efficiency of MUSIC algorithm,” IEEE Trans. Acoust., Speech Signal Process., vol. 36, no. 4, pp. 532–544, Apr. 1988.

[20] B. P. Ng, “A MUSIC approach for estimation of directions of arrival of multiple narrowband and broadband sources,” Signal Process., vol. 40, pp. 319–323, 1994.

[21] X. Mestre, M.Á. Lagunas, Modified subspace algorithms for DOA Estimation with large arrays, IEEE Trans. Signal Process, vol56, no 2, pp598–614, 2008.

[22] Veerendra, Md.Bakhar and Vani.R.M, “Implementation and Optimization of Modified MUSIC algorithm for High Resolution DOA Estimation”, Proc. in IEEE Int. Microwave and RF Conf., pp 190-193, December 2014.

[23] M. Haardt, A.N. Josef, Unitary ESPRIT: How to obtain increased estimation accuracy with a reduced computational burden, IEEE Trans. Signal Process, vol.43, no.5,pp.431232–1242, 1995.

[24] Paulaj, R. Roy, and T. Kailath, “Estimation of signal parameters via rotational invariance techniques-ESPRIT,” in Proc. 19th Asilomar Conf. Signals, Syst. Comput., Pacific Grove, CA, Nov. 1985, pp. 83–89.

[25] A. L. Swindlehurst and T. Kailath, “A performance analysis of subspace-based methods in the presence of model errors, Part I: The MUSIC algorithm,” IEEE Trans. Signal Processing, vol. 40, pp. 1758–1774, 1992.

[26] M. Wax and T. Kailath, “Detection of signals by information theoretic criteria,” IEEE Trans. Acoust., Speech, Signal Process., vol. 33, no. 2, pp. 387–392, Apr. 1985.

[27]A. B.Gershman, “Improved DOA estimation via pseudorandom resampling of spatial spectrum,” IEEE Signal Process. Lett.,Vol. 4, No. 2, 54–57, 1997.

[28]Wen-Jun Zeng, “lp-MUSIC: Robust Direction-of-Arrival Estimator for Impulsive Noise Environments” IEEE Trans. Signal Processing, vol. 61, no. 17, 4296- 4308, 2013.

[29] J. E. Evans, J. R. Johnson, and D. F. Sun, “Application of advanced signal processing technique to angle of arrival estimation in ACT navigation and surveillance systems,” MIT Lincoln Lab., Lexington, USA, MA, Tech. Rep. 582, 1982.

[30]Zhang, J. X., Christensen, M. G., Dahl, J., Jensen, S. H., & Moonen, M, “Robust Implementation of the MUSIC algorithm, Proc. of the IEEE International Conference on Acoustics, Speech and Signal, 3037-3040, 2009.

[31]S. U. Pillai and B. H. Kwon, “Forward/backward spatial smoothing techniques for coherent signal identification,” IEEE Trans. Acoust., Speech, Signal Process., vol. 37, no. 1, pp. 8–15, Jan. 1989.

[32]A.Nicholas, J.W.Patrick, Estimating principal components of large covariance matrices using the Nyström method, IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Prague,Czech Republic, 3784–3787, 2011.

[33] F. M. Han and X. D. Zhang, “An ESPRIT-like algorithm for coherent DOA estimation,” IEEE Antennas Wireless Propag. Lett., vol. 4, pp. 443–446, 2005.